

**A handbook of activities  
in  
MATHEMATICS  
for children**

by  
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## **PREFACE**

Mathematics, as we all know, is being used extensively by scientists, engineers and technologists and this subject has a special place in any curriculum. But most children have a phobia for mathematics perhaps because of the way the subject is being taught.

Maths Lab should not just be a place where children see a demo of readymade models or use models made professionally by others. It should be a place where there is a lot of hands-on activity. Furthermore, activities should aim at sharpening the intellect of the child apart from teaching syllabus-based topics.

This handbook has 4 parts – ARITHMETIC, ALGEBRA, GEOMETRY and MISCELLANEOUS each part containing activities on almost all major topics which the students study in classes 5 to 9. The book is written topicwise and not classwise. Students have to choose the topics relevant to their standard. It has been prepared in such a way that the student can easily read, understand and do the activities with little or no help from the teacher. Materials suggested are commonplace low cost materials easily available in shops.

Every activity is followed by an ACTIVITY SHEET with spaces for writing notes and sticking the models made. Challenging questions are also given in some Activity sheets.

I do believe that the book will go a long way in making mathematics an interesting and enjoyable subject. Suggestions for improvement are welcome.

**Prof. S. SUNDARARAJ**

## **GUIDELINES TO STUDENTS**

- ✦ Use always a good pair of scissors or a sharp cutter
- ✦ Do not tear the sheets with your hand
- ✦ Use always a ruler and pencil to draw lines
- ✦ Keep your Geometry Box ready
- ✦ Use glue sticks for sticking models on the Activity sheets.

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## ARITHMETIC

### Activity 1 LARGE NUMBERS AND PLACE VALUES

**Materials:** Numbered cards (0 to 9) 3 sets

- ✓ Shuffle the cards and pick 6 cards at random.
- ✓ Arrange the cards so as to get the largest possible 6-digit number. Note down this number in the Activity book. Now form the smallest 6-digit number with the **same** 6 cards and note it down.
- ✓ At the next round, shuffle the cards again and pick 8 cards and form the biggest and smallest possible numbers. Note the numbers in the AS.

Decimal Number System is made up of 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called NUMERALS.

The value of digit depends on its position in a number. The value of the places increases to the left by a factor of 10 and decreases to the right by a factor of 10.

For example, in the number 52186, the face values of 5, 2, 1, 8 and 6 are respectively 5, 2, 1, 8 and 6 while their place values are 50000, 2000, 100, 80 and 6.

## Questions/Challenges

Fill in the blanks in the following Multiplication Tables.

a.

x	7	4	8
5			
6			
9			

b.

x		11	20
3	30		
2			
5			

c.

x	3	4	10
4			
5			
20			

d.

x	9	4	2
	54		
		36	
			22

e.

x	12	2	
		20	
	96		
6			60

f.

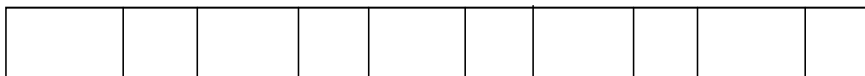
x			
	64	24	16
	32	12	8
	56	21	14

## Activity 2 EXPANDED FORM OF LARGE NUMBERS

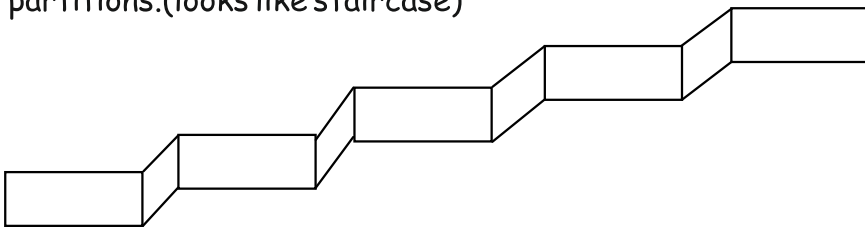
(place value snake)

**Materials:** Thick paper strips of size 30 X 3 cm (2)

**Procedure**

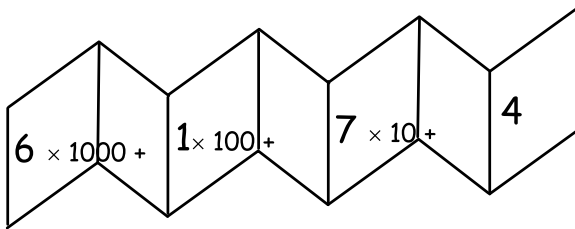


Fold alternately first up and then down at all the partitions.(looks like staircase)



Now digits have to be written over all the folds and the expansion of each digit to be written under the fold so that in the folded position, only the digits will appear. When the strip is unfolded slowly one at a time, you should see the expansions.

Example:



(note that 6 is on top of the fold and ' $\times 1000 +$ ' is underneath the fold)

Make 2 such 'snakes' for 2 numbers.

### Activity 3 PAPER CUP MODEL FOR PLACE VALUES

**Materials:** 8 small white, plain, stiff paper cups of height about 8 cm. To get a gap between 2 cups when they are stacked, a small piece of sponge may be stuck to the bottom of all the cups.

#### **Procedure**

- At the edge of one cup, numbers 0, 1, 2, ..., 9 should be written sideways all around the edge at equidistant intervals. At the edge of the 2nd cup numbers 00, 10, 20, 30 ... 90 have to be written. (See photos below). This has to be repeated till 00000000, 10000000, etc

#### **How to use?**

When the cups are stacked in order from ones place to millions place as shown in the second picture, 8-digit numbers appear. To find place values of digits, just remove suitable cups and see the hidden number inside.



After making the model, answer the questions given in the Activity sheet.

### Challenge !

Find  $y$ ,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ ,  $f$ ,  $g$ ,  $h$  and  $i$  from the set of numbers  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$1 \times y + a = 9$$

$$12 \times y + b = 98$$

$$123 \times y + c = 987$$

$$124 \times y + d = 9876$$

$$12345 \times y + e = 98765$$

$$123456 \times y + f = 987654$$

$$1234567 \times y + g = 9876543$$

$$12345678 \times y + h = 98765432$$

$$123456789 \times y + i = 987654321$$

#### Activity 4 ARITHMETIC OPERATIONS ON NUMBERS USING TILES

*Materials:* 8 Cardboard strips of size 20 X 3 cm, scissors

##### *Procedure*

Cut each strip into square pieces (tiles). Each tile is assumed to have area 1 and it represents number 1. They are called UNIT TILES. Write a big plus sign on one side and a big minus sign on the opposite side. Positive side represents 1 and negative side is - 1. Addition is done by simply placing tiles in a row side by side.

Furthermore, one positive tile together with a negative tile will give 0. Such a pair of tiles is called 'zero pair'.

For example, to add 4 and - 5 place 4 positive side tiles and 5 negative side tiles side by side. Remove all the zero pairs. You will be left with one 'minus tile' and hence  $4 + (-5) = -1$ .

Practice this by writing your own problems.

Subtraction. To calculate  $5 - (-2)$

Place 5 positive side tiles and 2 negative side tiles side by side. Since it is subtraction, the second set of 2 tiles should be *flipped*, leaving all positive tiles. Answer will be 7.

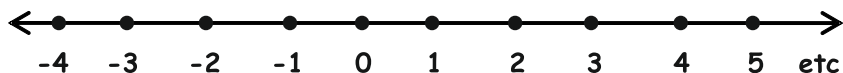
Do the problems given in the Activity sheet.

## Activity 5 INTRODUCTION TO NUMBER LINE

This is an *outdoor* activity. It is best done as a group with the help of a teacher in a corridor or on a playground.

*Materials:* Chalk pieces

**Introduction.** Numbers can be represented geometrically by points on a line. Take one point on the line as 0 and positive numbers on one side of 0 (to the right usually), negatives on the opposite side of 0 (usually to the left). Mark equidistant points on the line to represent 1, 2, 3, - 1, - 2 etc.



Every point on the line represents some number or the other. The gaps in the line contain other types of numbers.

### Procedure

- A long line has to be drawn on the ground on which small circles or squares, equally spaced, to be drawn for students to stand on.
- Students are made to stand on the line side by side facing the teacher.
- Students should be told that as they move to their left, numbers get bigger and bigger and as they move right, value gets smaller. Now, only one student has to stand on the line while all others watch. He or she has to walk to the right or to the left and practice, 'Add 3 to 1', 'Add 2 to - 4', 'Add - 3 to 3' etc. Every

student should get a chance.

- Adding is *turning left and walking forward* and subtracting is *walking backward*. Students have to practice adding/subtracting positive and negative numbers. Examples:  $3 + 4$ ;  $3 - 4$ ;  $- 3 + 4$ ;  $- 3 - 4$ ;  $4 + (-2)$ ;  $(- 4) + (- 3)$ ;
- Now activity sheet has to be completed.

### **Activity 6 INTEGER OPERATIONS I (game)**

This game has to be played by pairs of students.

Materials:

- 1 Cardboard containing numbers as shown in the Activity Sheet to be used as game board. (GAME BOARD is printed on the Activity sheet)
- 2 6 white dice and 6 black dice in a box.
- 3 2 small counters of different colors, one for each player.

**Procedure:**

White dice indicate positive integers and black dice indicate negative integers.

- Every player will place his/her counter at zero.
- First player will take out two dice at a time from the box without looking at the colors and roll them.
- After every throw, the player has to multiply the

number of dots appearing on the top faces remembering that dots on the white die is positive while those on the black die is negative.

- If the product is a positive integer then the player will move his counter towards 60; if the product is a negative integer then the player will move his counter towards -60.
- During the game, counters will be moving forward **and** backward.
- Only horizontal motion is allowed. No vertical or diagonal motion allowed.
- Whoever reaches 60 or - 60 or a number nearest to 60 or - 60 is the winner.

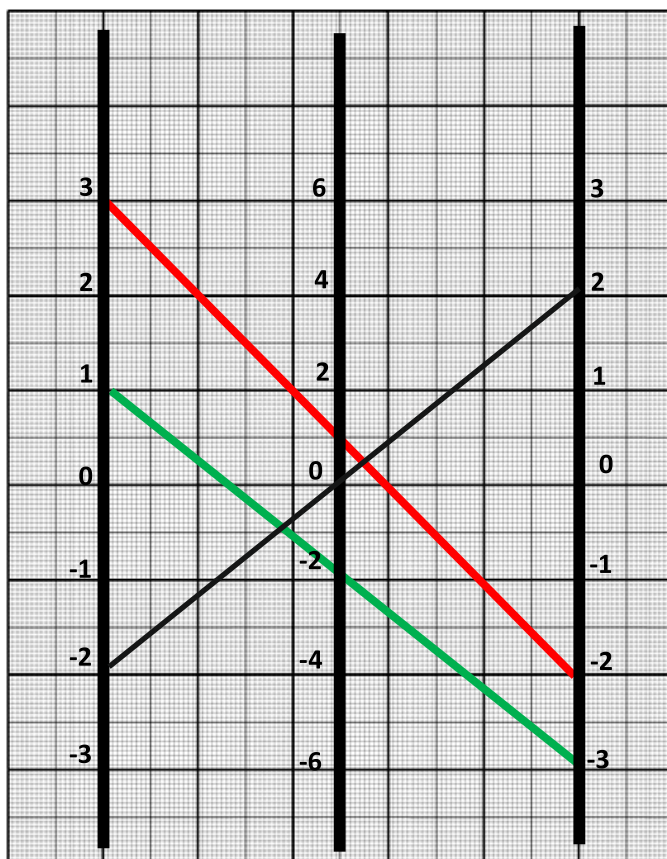
## Activity 7 INTEGER OPERATIONS II (NOMOGRAPH)

*Nomograph* is a form of graph when 3 quantities are to be connected to produce a third quantity. The basic concept used in all nomographs is similar triangles.

Materials: Ordinary graph sheet of A4 size, ruler

Procedure:

Draw 3 equidistant parallel lines vertically on the graph sheet as shown below.



Write the numbers 0, 1, 3, - 1, - 2 etc on the first and third lines and on the middle line, take half the scale and write 0, 2, 4, - 4, - 6 etc.

Nomograph is ready!

*How to use?*

Suppose you want to add 3 and - 2, take a ruler and connect 3 on the first line with - 2 on the 3rd line with a ruler and see where the ruler meets the middle line. It meets at 1. Therefore answer is 1.

Similarly to add -2 and 2 connect - 2 on the first line with 2 on the 3rd line. Answer 0 appears in the middle line.

Subtraction: Find  $(-2) - (-3)$

Connect - 2 in the MIDDLE LINE with - 3 on the THIRD LINE with a ruler and see where the ruler meets the FIRST LINE. Answer appears on the FIRST line.

## **Activity 8 FACTORS AND PRIME NUMBERS**

*Materials:* 40 square tokens (plain) in a box/bag, color pencils of 2 colors.

*Procedure*

- Assume the area of each token as 1 square unit. Pick a handful of tokens from the box without counting. Try to make a rectangle using all the tokens in your hand like floor tiles. Of course one long rectangle of breadth 1 unit can always be made. See whether you

can make a rectangle other than this. If you succeed, note the length and breadth of your rectangle. For example, with 20 tokens you could make a 5 X 4 rectangle or a 10 X 2 rectangle. Here the numbers 5, 4, 10, 2 are called **FACTORS** or **DIVISORS** of 20. Even 20 and 1 are factors of 20 (trivial factors).

- Now replace the tokens and make as many rectangles as possible with all the 40 tokens and find all the factors of 40.
- Prime number is a number having just two factors, 1 and itself. 1 is not a prime number and 2 is the first prime number. The primes between 1 and 10 are 2, 3, 5, 7.
- Do the activity given in the Activity Sheet.

## **Activity 9 SIEVE OF ERATOSTHENES**

*Materials:* Activity sheet only

*Procedure*

Write all the natural numbers starting from 2 up to 103 in 6 columns printed on the Activity Sheet.

- ✓ Circle 2 and strike off all multiples of 2. Numbers in first, third and fifth columns will go off.
- ✓ Circle 3 and strike off all multiples of 3. Now numbers in the second and 5th columns will go off.

- ✓ Multiples of 4 are even, they are already crossed out. So check for 5 and 7 strike the numbers divisible by them.

Now we are left with prime numbers.

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55
56	57	58	59	60	61
62	63	64	65	66	67
68	69	70	71	72	73
74	75	76	77	78	79
80	81	82	83	84	85
86	87	88	89	90	91
92	93	94	95	96	97
98	99	100	101	102	103

## **Activity 10 INTRODUCING HCF through an activity**

*Materials:* 3 Ribbons of different colors of lengths 24cm, 36cm, 48cm and a thin stick (which can be cut easily using scissors) of length 15cm ; ruler, scissors

### *Procedure*

- You are going to measure the lengths of all the ribbons using the given stick of unknown length. It will not be possible to measure any ribbon exactly with the stick.
- Now cut 1 cm of the stick and measure again. It will not work. Cut another 1 cm and measure. Even now it will not work. After 2 attempts the 3<sup>rd</sup> cut will work. You will be able to measure all the ribbons exactly. Measure the stick and you will get 12 cm. This number is the HCF of 24, 36, 48.

## **Activity 11 INTRODUCING LCM through an activity**

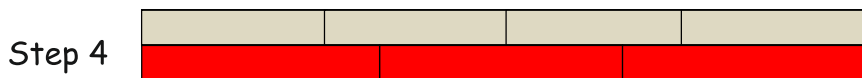
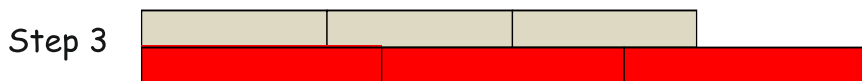
*Materials:* 5 sets of stiff cardboard strips of width about 1.5cm, each set consisting of strips of lengths 1.5cm, 3cm, 4.5cm, 6cm, 7.5cm, 9cm, 10.5cm, 12cm, 13.5cm and 15cm in different colors. (50 strips in all)

The smallest strip (length 1.5cm) is to be called a *1-unit strip*, the next in size

(length 3cm) to be called *2-unit strip*, the third (length 4.5cm) to be called *3-unit strip* and so on. The longest will be the *10-unit strip*.

*Illustration* Find the LCM of 6 and 8

Select one 6-unit strip and one 8-unit strip. Place the 2 strips one below the other.



Now place another 6-unit strip by the side of the first at the top and another

8-unit strip at the bottom row. See if the rows match. Here they don't match. Now keep placing strips until the rows match. The common length obtained will be the LCM. In this case the common length is 24 and hence the LCM is 24

## Activity 12 CALCULATION OF LCM USING TIMES TABLES

*Materials:* Grid sheet having multiplication tables

*Procedure*

*Illustration.* Find the LCM of 4 and 9

Run your left forefinger along the Row 4 and your right forefinger along Row 9. When you get a common number, stop. You will find that the first common entry is 36. Therefore the LCM is 36.

MULTIPLICATION CHART TO 10x10										
1	2	3	4	5	6	7	8	9	10	
2	4	6	8	10	12	14	16	18	20	
3	6	9	12	15	18	21	24	27	30	
4	8	12	16	20	24	28	32	36	40	
5	10	15	20	25	30	35	40	45	50	
6	12	18	24	30	36	42	48	54	60	
7	14	21	28	35	42	49	56	63	70	
8	16	24	32	40	48	56	64	72	80	
9	18	27	36	45	54	63	72	81	90	
10	20	30	40	50	60	70	80	90	100	

Now go to the Activity sheet and complete the activity.

### Activity 13 FRACTIONS AND THEIR COMPARISON

**Materials:** 6 craft paper strips of size 15 X 2 cm in 6 colors, gluestick.

- ✓ Fold the first strip into 2 equal parts.
- ✓ Each part represents  $\frac{1}{2}$ . So write  $\frac{1}{2}$  on each part.
- ✓ Fold the second strip into 3 equal parts.
- ✓ Each part represents  $\frac{1}{3}$ . Write  $\frac{1}{3}$  on each part.
- ✓ Fold the third strip into 4 equal parts. Write on  $\frac{1}{4}$  each part.

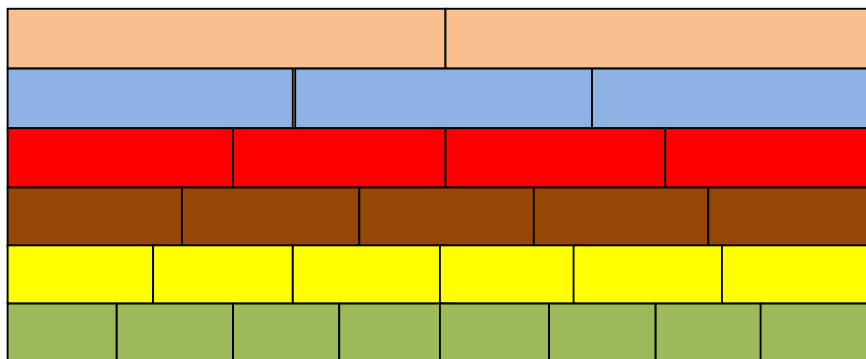
Similarly prepare strips having 5, 6 & 8 parts. (skip 7)

To compare  $\frac{1}{2}$  and  $\frac{1}{3}$ , fold the parts showing  $\frac{1}{2}$  and  $\frac{1}{3}$ .

Superimpose one on the other and see which is longer.

To compare  $\frac{1}{3}$  and  $\frac{1}{5}$  superimpose  $\frac{1}{3}$  and  $\frac{1}{5}$  and see which strip is longer.

Now arrange the strips in descending order and stick them in the Activity sheet. This is usually called the FRACTION WALL.



Go to the Activity sheet and answer the questions given in it

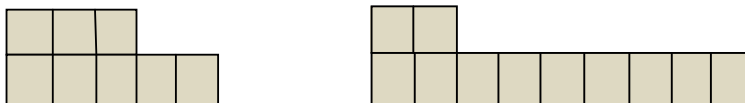
**Activity 14** In this activity **ADDITION AND SUBTRACTION OF FRACTIONS** are done using Cuisenaire strips.

*Materials:* Cuisenaire strips

Cuisenaire strips can be made very easily. They are just 12 strips of card of lengths 1 unit, 2 units, 3 units etc upto 12 or 15 units. Partitions are shown on all the strips.

### Representation of fractions using Cuisenaire strips

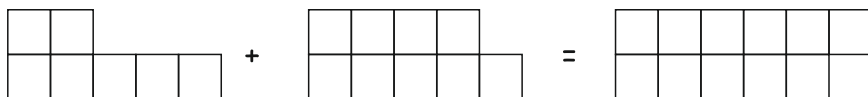
The following diagram shows  $\frac{3}{5}$  and  $\frac{2}{9}$ . Place strips one below the other just like numerator and denominator.



### How to add fractions?

*Illustration 1* Add  $\frac{2}{5}$  and  $\frac{4}{5}$

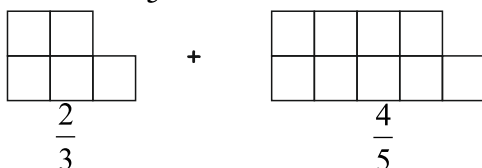
The following diagram is self-explanatory.



*Illustration 2*

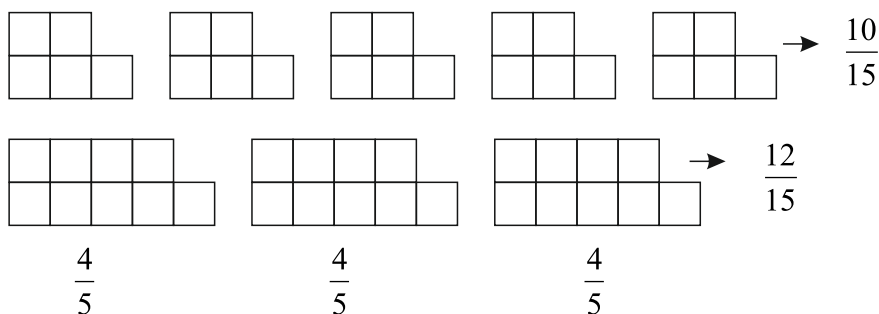
Add  $\frac{2}{3}$  and  $\frac{4}{5}$

Make  $\frac{2}{3}$  and  $\frac{4}{5}$  using suitable strips.



To add the given fractions, the denominators of both should be made equal. So, place more 5-length strips next to the denominator strip of the first fraction and place 3-length strips next to the second fraction till the 'denominator strips' form equal trains.

**Important:** Each time a strip is placed in the denominator, a strip equal to the numerator strip must be placed as shown below in order to get equivalent fractions. Add the numerator of both fractions.



$$\frac{10 + 12}{15} = \frac{22}{15}$$

Now answer the questions given in the Activity sheet.

### Activity 15 ADDITION AND SUBTRACTION OF FRACTIONS using tracing paper.

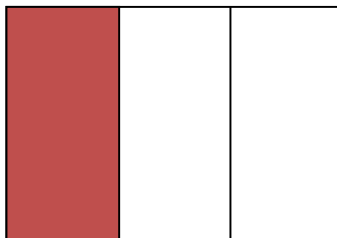
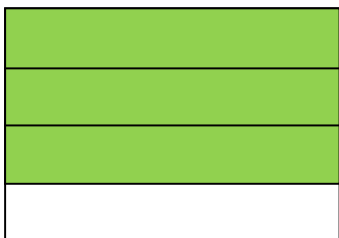
**Materials:** 10 X 7 cm tracing papers (8) (A4 size cut into 8 parts); coloring material.

**Procedure.**

Calculate  $\frac{3}{4} + \frac{1}{3}$  and  $\frac{3}{4} - \frac{1}{3}$

Fold one tracing sheet lengthwise into 4 equal parts and

another sheet breadthwise into 3 equal parts. Unfold and draw dark lines along all the creases. Now show  $\frac{3}{4}$  on the first sheet by shading with a color and  $\frac{1}{3}$  on the 2nd sheet by shading in another color.



Now superimpose the sheets and count the number of boxes formed. It is 12.

Out of 12, number of green boxes is 9 and so  $\frac{3}{4}$  is same as  $\frac{9}{12}$ . Similarly number of brown boxes will be 4 out of 12 and so  $\frac{1}{3} = \frac{4}{12}$ .

Adding these fractions, answer is  $\frac{13}{12}$ . Subtraction gives  $\frac{5}{12}$ .

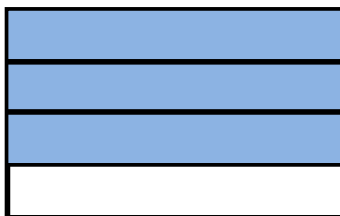
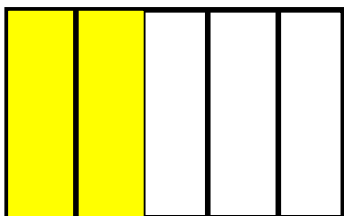
### **Multiplication of fractions by paper folding.**

*Materials:* 10 X 7 cm tracing sheets (4)

*Procedure*

Multiply  $\frac{2}{5}$  by  $\frac{3}{4}$

**Step 1** Fold one sheet into 5 equal parts breadthwise and the second sheet into 4 equal parts lengthwise (because 5 and 4 are the denominators)



There are 20 boxes. Hence denominator of the product is 20.

Step 2 Take the first sheet and color  $\frac{2}{5}^{\text{th}}$  part yellow.

Step 3 Take the 2nd sheet and color  $\frac{3}{4}^{\text{th}}$  part blue.

Step 4 Superimpose the sheets and count the number of boxes which are of both colors (looks green). This will be the numerator.

## Activity 16 This activity is on SQUARES AND SQUARE ROOTS

**Materials:** Square tokens of size about 1.5cm x 1.5cm (40)

### Procedure

Do you know how to build a square shape using tokens?

- ✓ Take 3 tokens and try to make a square using them. It will not be possible. Then take 4 tokens and try to make a square. It should be possible to make one. Next try to make a square with 5, 6 etc tokens. Find out how many tokens will make a square.

Note down in the Activity sheet.

- ✓ Observe the number of tokens used along the side.

Note the number in the activity sheet.

- ✓ Is there any relation between the two numbers?
- ✓ Write the conclusion.

**Short cut method of finding the square of a number ending with 5**

**Example** Find the square of 45

- ✓ Draw 4 boxes.

--	--	--	--

- ✓ Write 25 in the last 2 boxes.

		2	5
--	--	---	---

- ✓ Write  $4 \times (4 + 1) = 4 \times 5 = 20$  in the first 2 boxes.  
So,  $45^2 = 2025$

2	0	2	5
---	---	---	---

## Activity 17 Finding SQUARE ROOTS using a strip of paper

*Materials:* A strip of ruled sheet of width 3 cm

*Procedure:*

Example Find the square root of 25.

Cut a strip of paper of width about 3 cm from a ruled sheet. Length should be 25 boxes. (if square root of 49 is required, then take 49 boxes)

With a pair of scissors, start cutting from one end. First cut one box and keep it on the table. Cut 3 boxes and keep it with the previous one. Next cut 5 boxes, then 7 boxes. The remaining strip will have 9 boxes on it. The total number of pieces, namely 5, is the square root of 25. *Note that you have to cut odd number of boxes always.*

## Activity 18 This activity is on CUBE ROOTS

This activity will give you a quick way to find cube roots. It is called the INSPECTION METHOD.

*To use this method you have to notice the patterns in the cubes of the first 9 digits.*

$1^3$	1
$2^3$	8
$3^3$	27
$4^3$	64
$5^3$	125
$6^3$	216
$7^3$	343
$8^3$	512
$9^3$	729

*Note that the cube root of a number ending in 0, 1, 4, 5, 6 and 9 always end in the same digits. Also, cube root of numbers ending in 2 end in 8 and vice versa and cube root of a number ending in 3 end in 7 and vice versa.*

To find the cube root remember these steps:

**Step 1** Look at the ending digit and decide about the last digit of the answer.

**Step 2** Leave the last 3 digits (always) and take the number that remains.

**Step 3** Find 2 cube numbers between which this number lies, from the cubes table. Take the smaller one and write down the answer.

**Illustration 1.** Find the cube root of 1728

The last digit is 8 and so the last digit of the answer must be 2 from the above table. When we leave out 728 (step 2) we get 1. In the table, 1 is found between  $1^3$  and  $2^3$ . Choosing 1, the required cube root is 12.

**Illustration 2** Find the cube root of 39304

Inspect the last digit. Since it is 4, the units place of the answer will be 4. Leave the last 3 digits. We are left with 39 only. 39 lies between  $3^3$  and  $4^3$  (see table and decide). Choose 3 and get the answer as 34.

*This method will hold good for all cube numbers, however big it is.*

**Cube roots by successive subtraction****Illustration** Find the cube root of 216

$$\begin{array}{r} 216 \\ - 6 \\ \hline 210 \\ - 12 \\ \hline 198 \\ - 24 \\ \hline 174 \\ - 42 \\ \hline 132 \\ - 66 \\ \hline 66 \\ - 66 \\ \hline \end{array}$$

As there are 6 subtractions, the cube root is 6.

Note that the numbers subtracted are 6, 12, 24, 42, 66 etc.

## **Activity 19 This activity is on VARIATION (unitary method)**

*Materials:* Empty plastic water bottle. 3 Bottle caps with 1 small hole in the first, 2 same sized holes in the 2<sup>nd</sup> and 3 holes in the 3<sup>rd</sup> cap. Stop watch/timer and water in a can/mug.

### *Procedure*

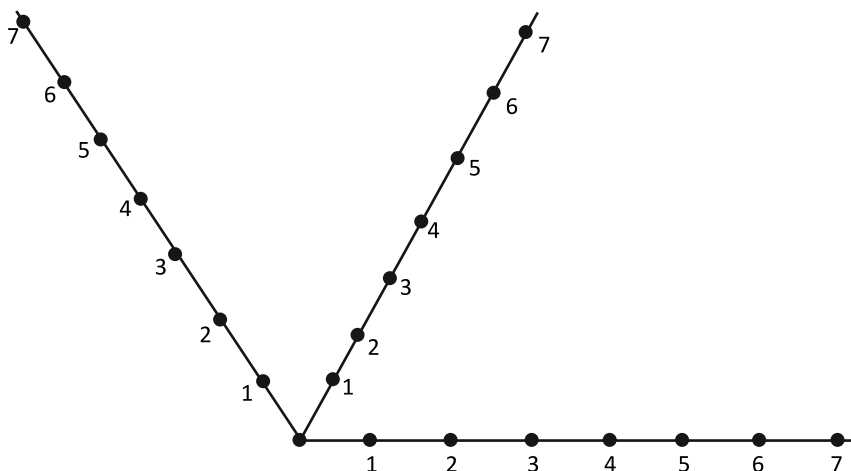
- ✓ The bottle should be filled with water and covered with the cap with 1 hole.
- ✓ Now, start the stop watch and invert the bottle and allow the water to empty into a mug
- ✓ Repeat the above step by using caps with 2 holes and 3 holes. Make a note of the time for each trial in the activity sheet.
- ✓ Observe the values in the table. It will be seen that the time taken to empty the bottle decreases with the increase in the number of holes. This is an example of inverse variation.

Using the above data, it is possible to work problems on unitary method. Go to Activity Sheet.

## Activity 20 This activity is on TIME AND WORK

Problems on Time and Work can be solved by what is called **nomograph**. In this Activity, you will learn this method.

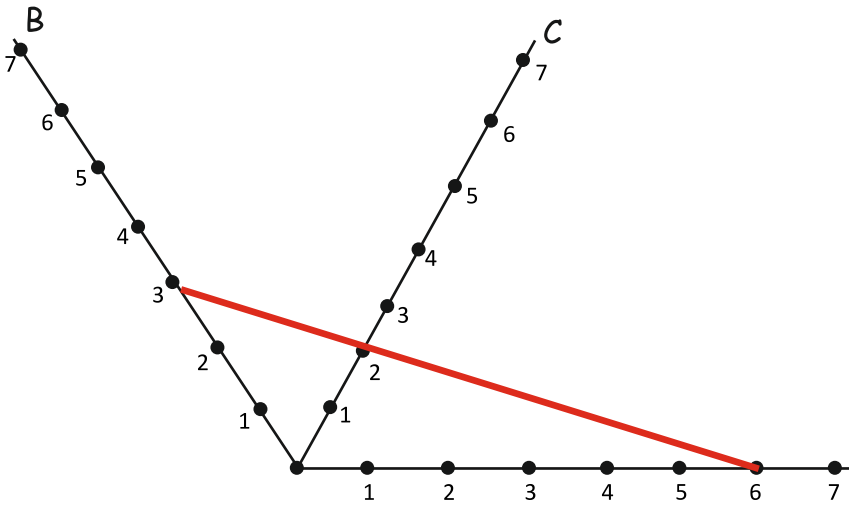
*Materials:* Geometry Box; thin stick or ruler and pencil



*Procedure:* Take a sheet of blank paper. Draw a line OA of any convenient length. Draw a line OB = OA such that angle AOB =  $60^\circ$ . Draw OC = OA such that angle AOC =  $120^\circ$ . Use protractor and measure angles accurately.

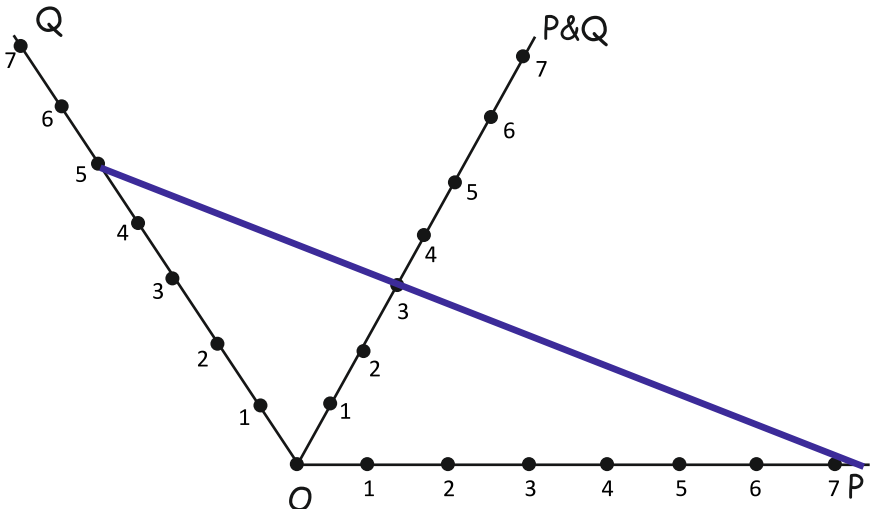
**Illustration 1** Arun can paint a building in 6 days while Chetan can do the same job in 3 days. If they do the job together how many days will they take to paint the building?

*Procedure:* Let OA represent Arun's job and OC, Chetan's. Take a thin stick and place it on the nomograph in such a way that it touches the line OA at 6 and line OC at 3.



You will see that the stick intersects the middle line at '2'.  
Therefore the answer is 2 days.

**Illustration 2** P and Q together can build a wall in 3 days.  
If Q alone requires 5 days to build the same wall, how long  
does P require to build it alone?



Connect 3 on the middle line with 5 on the line OQ and see where the stick cuts OP.

Since it cuts OP at 7.5, the answer is 7.5 days.

Choose similar questions from your textbook and try to get the answers using nomograph.

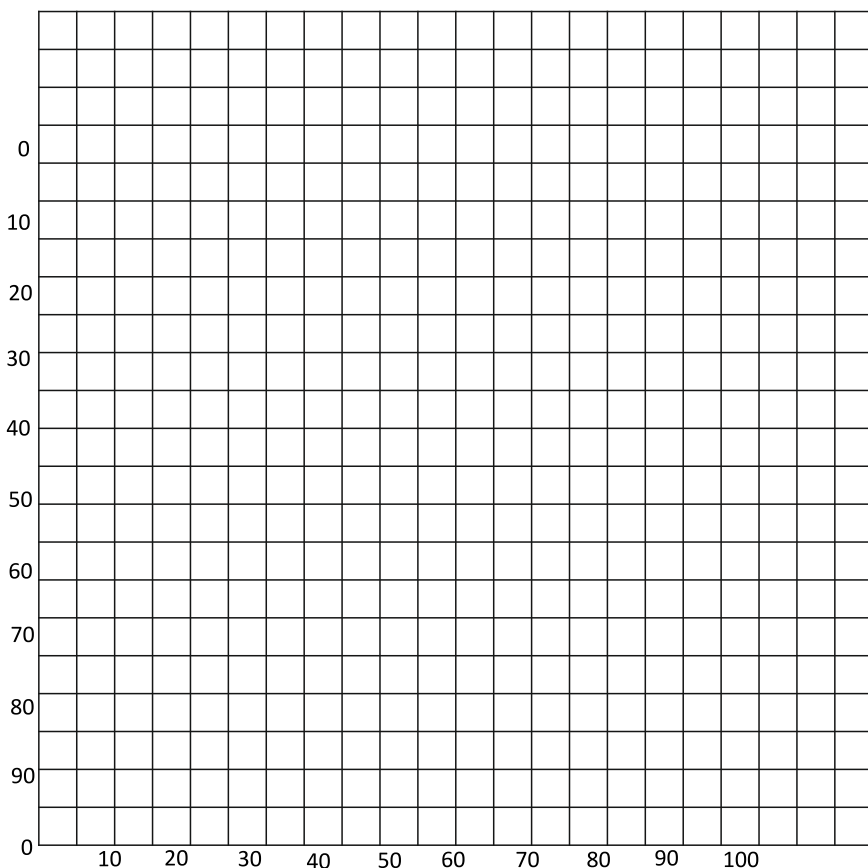
*Remember, the answers will only be approximate. By subdividing the scales given above it is possible to obtain more accurate answers.*

## Activity 21 In this activity, you will learn to make a PERCENTAGE CALCULATOR

**Materials:** A graph sheet, long ruler, an L-angle.

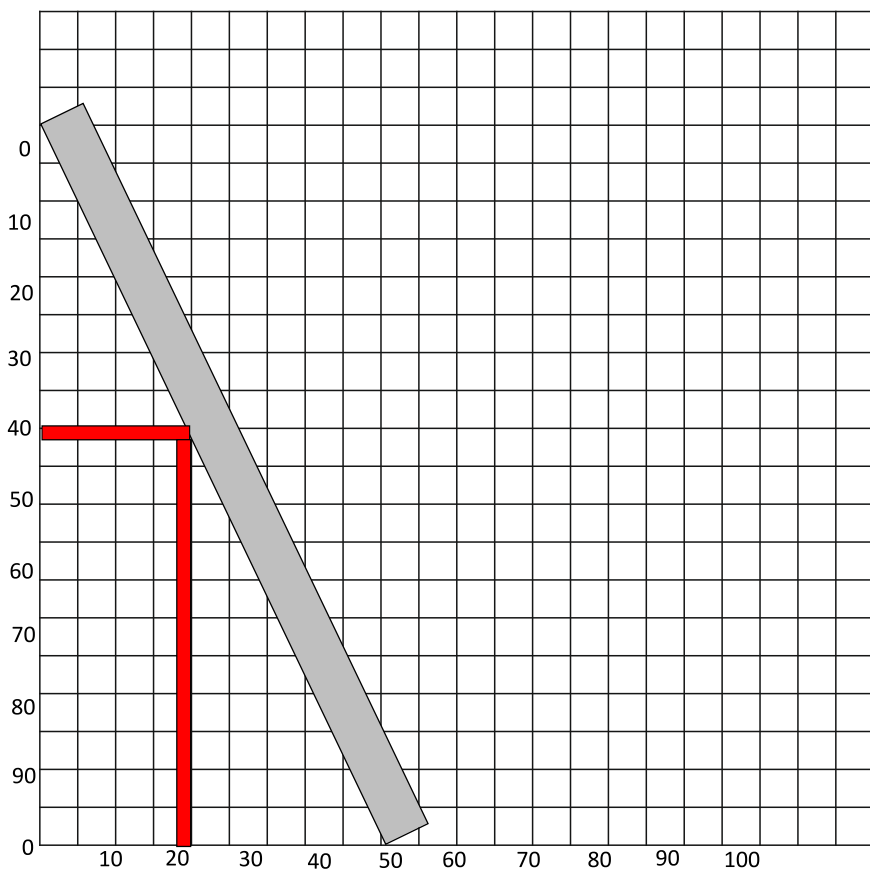
**Procedure:** L-angle can be made by joining 2 sticks or cardboard strips of about 28 cm each, at right angles.

On the graph sheet, mark 0, 10, 20, 30 . . . . . 100 as shown below.



**Illustration 1** Find 40% of 50

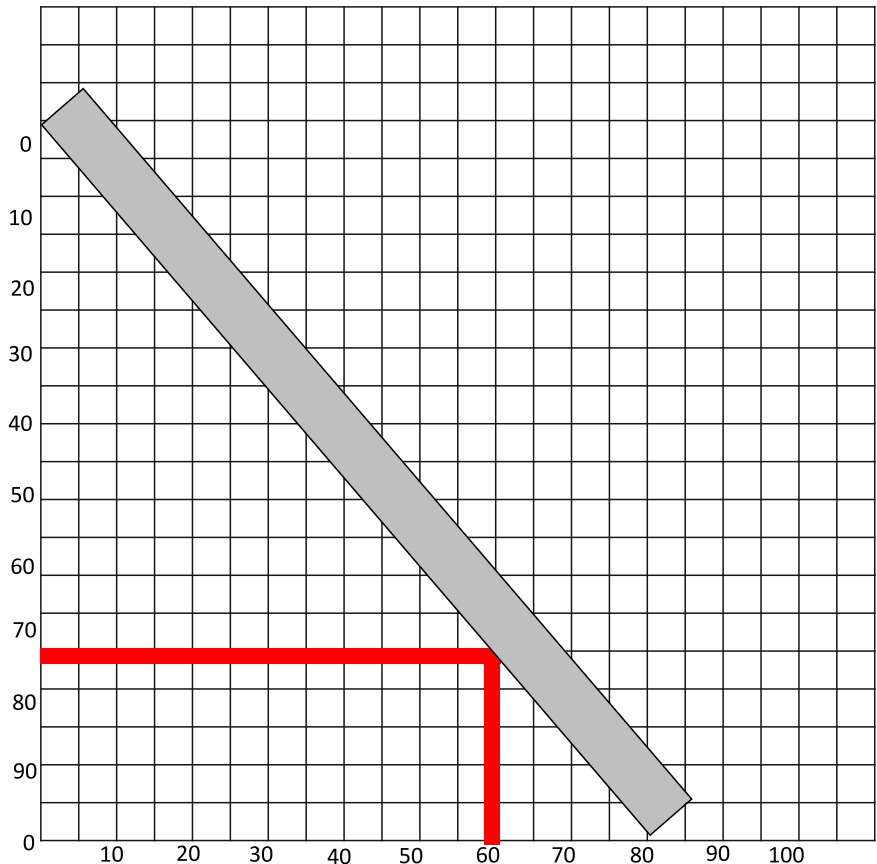
**Method:** Place a ruler connecting '0' on the y axis with '50' on the x axis. Holding the ruler tightly in this position, place the L-angle as shown below in such a way that one leg is at 40 and the right angle corner just touches the ruler. The point B where the vertical leg of the L-angle meets the x axis is at 20. Therefore 40% of 50 is 20. Answer appears on the x axis.



**Illustration 2** What percent of 80 is 60?

Method: Place the ruler connecting '0' on the y axis and 80 on the x axis.

Now move the set square till one leg passes through 60 on the x axis and the right angled corner just touches the ruler. Read the answer on the y axis as 75



# ALGEBRA

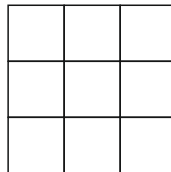
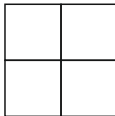
## Activity 22 RECOGNIZING AND GENERALIZING PATTERNS

*Materials:* 25 cards (TILES) of size 2 X 2 cm - 4 white, 5 green, 7 red, 9 yellow

### *Procedure*

Take the Activity sheet, stick the tiles as shown below and tabulate your observations.

Tiles to be assumed of area 1 sq.unit are to be pasted in such a way that they form squares, small to big. The first will have one white tile and each side is 1 unit. Then add 3 tiles to get the 2<sup>nd</sup> square, 5 tiles to get the 3<sup>rd</sup> square so on.



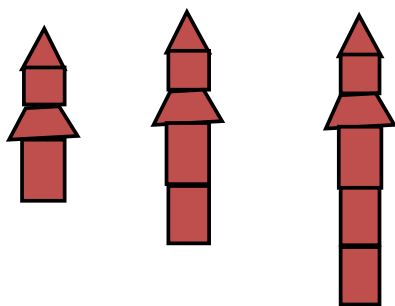
After the 4<sup>th</sup> square, you have to guess the number of tiles and enter the number in the table. Using the 2<sup>nd</sup> table it is possible to find the pattern and write down the general term.

## Activity 23 RECOGNIZING AND GENERALIZING PATTERNS IN A ROCKET

*Materials:* Cards in the shape of isosceles triangle (4), square (4), trapezium (4) Plus 10 rectangular pieces. All pieces may be cut from 3 X 2 cm rectangular cards

### *Procedure*

The rocket pattern as shown below has to be made and affixed in the Activity sheet. The rocket is followed by puffs of smoke at the bottom. Then tabulate the number of pieces used to make each rocket pattern.



Go to Worksheet and complete the table given.

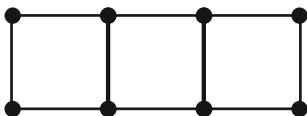
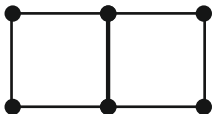
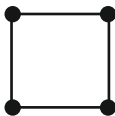
## Activity 24 TO OBSERVE THE PATTERN AND GENERALIZE.

*Materials:* 20 sticks of length 3 cm

Form patterns of squares using the sticks.

As soon as the first square is formed, start filling the given table

In the Activity sheet. Add 3 sticks to the same square, get 2 squares and enter in the table. This has to be continued till 5 squares are formed. Then questions given below have to be answered.

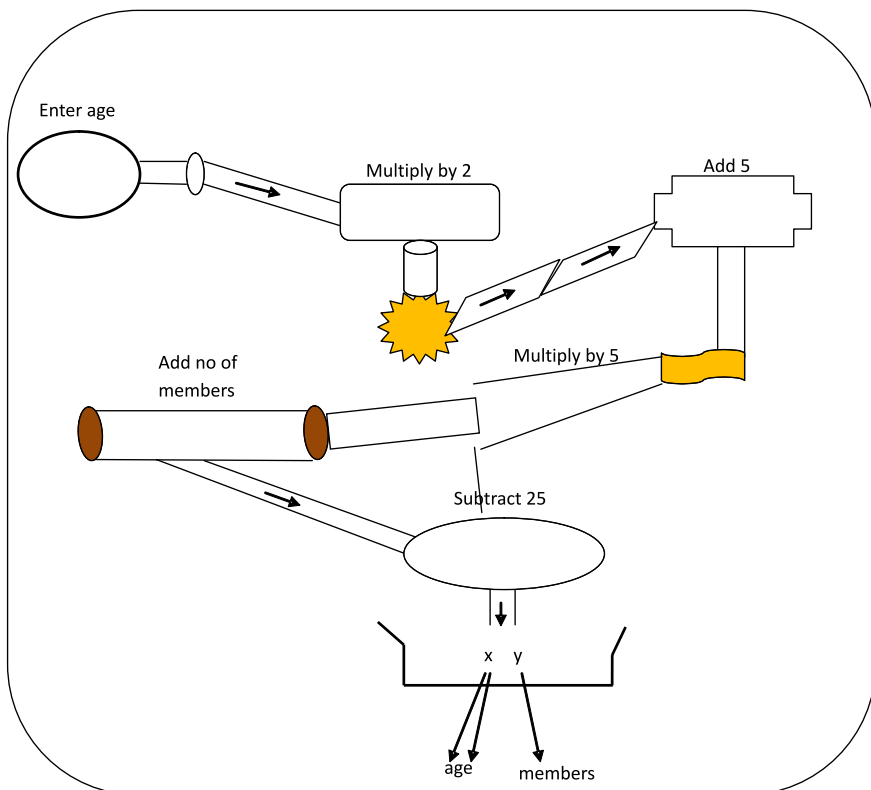


## Activity 25 AGE MACHINE (to practice writing algebraic expressions)

### Procedure

(1) Enter your age as  $x$  and the number of members of your family as  $y$ , get an algebraic expression at the end.

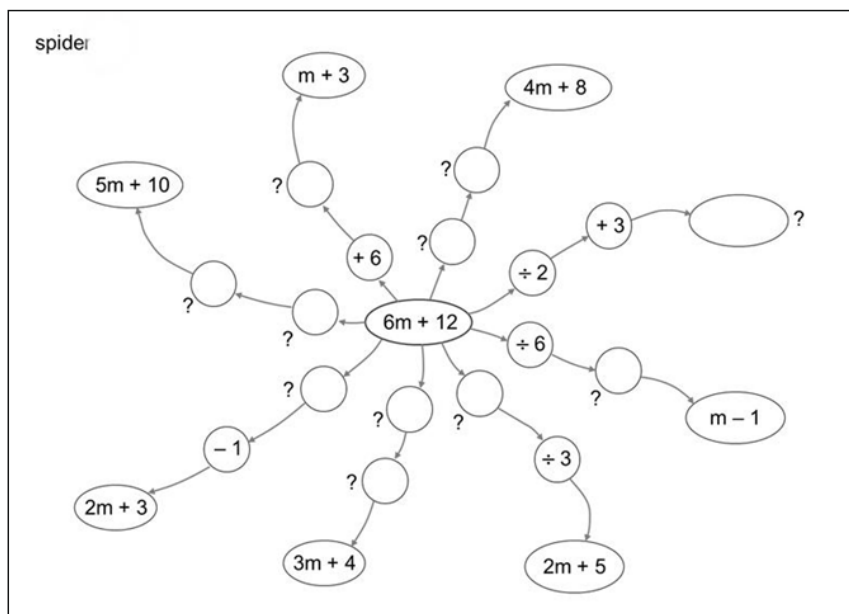
(2) You can act as the machine and play this as a mind reading game after deleting the last instruction.



## Activity 26 SPIDER GAME

In the following SPIDER, you have to fill in the blank circles with symbols, numbers or expressions containing variable  $m$ . For example, in the top 'leg' leading to  $m + 3$ , you have to enter

' $\div 6$ ' because,  $6m + 18$  divided by 6 is  $m + 3$













Activity 27 EVALUATION OF EXPRESSIONS I

Calculate the values of all the given expressions when  $a = 2$ ,  $b = 1$ ,  $c = 0$  and enter their values in the same order as a 4 X 4 square arrangement of numbers. If all your values are correct you will get a MAGIC SQUARE. *MAGIC SQUARE is a square arrangement of numbers such that the sum of numbers in any row or column or along a diagonal is the same. This sum is called magic constant.*

$2a - 2b$	$4a - 3b + 2c$	$a + 6b + c$	$2a + 3b$
$4a + 3c$	$a + 3b$	$5b - a$	$2b + c + a$
$8a - 13b + c$	$3a - 3b$	$4a + b$	$3ab - b$
$a^2 + 3b$	$5ab^2 - 3b$	$-$	$a^2b^2 + c^2$


## Activity 28 EVALUATION OF EXPRESSIONS II

For the following cross number puzzle, clues are given in the form of algebraic expressions. You have to evaluate the expressions for  $a = 4$ ,  $b = 5$  and  $c = 7$ .  $d$  is not given but as and when you get the values, go on filling the boxes which contain only  $a$ ,  $b$ ,  $c$ . Then it will be possible to find what  $d$  is. Once  $d$  is known you can complete the puzzle.

	1			2	3
4			5		
6		7			
		8		9	10
11	12			13	
14			15		

### ACROSS

1  $a + d$

2  $a + 2d$

4  $2d$

5  $5c + 6d$

6  $a^6$

8  $b^2c^3$

11  $abc$

13  $3(6a + b)$

14  $2(a + b)$

15  $ad$

### DOWN

1  $10(b + c)$

2  $ab$

3  $b + 8c$

4  $2d + c - b$

5  $3bd$

7  $abc^2$

9  $a^2c^2$

10  $3(b + 2c)$

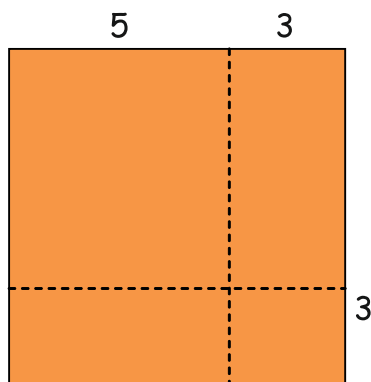
11  $d$

12  $a(b + c)$

## Activity 29 MAKING AND USING ALGEBRA TILES

*Materials:* KG card of size 8 X 8 cm (5 cards)

*Procedure:* Follow the cutting instructions given below.

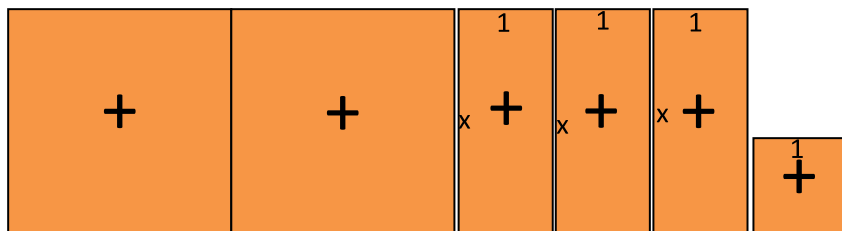


Draw 2 perpendicular lines as shown and cut the sheet into 4 pieces - one big square, one small square and 2 equal rectangles. Repeat the procedure for all the sheets. Your tiles will be ready.

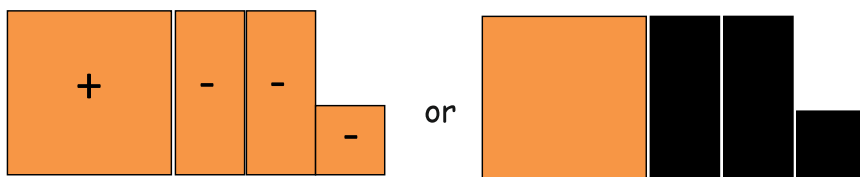
After you are done, you should have 20 tiles : 5 big squares, 10 rectangles and 5 small squares. One side of all the tiles is the 'positive side' and the other side is the 'negative side'. A big plus sign has to be written on one side and a big minus sign on the opposite side in the middle of the tile. ALTERNATIVELY COLOR ONE SIDE BLACK AND TAKE IT AS THE NEGATIVE SIDE.

Furthermore, the dimensions of sides have to be written on each tile. Write  $x$  as the side of each large tile, 1 as the side of each small tile. Mark the length of each rectangle as  $x$  and breadth as 1. Algebraic expressions are represented by

the areas of the tiles. Each large tile represents  $x^2$ , each rectangle represents  $x$  and each small square represents 1. Addition is done by placing tiles side by side. For example,  $2x^2 + 3x + 1$  is represented as shown below.



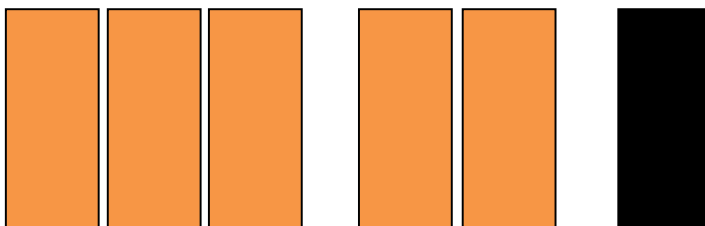
$x^2 - 2x - 1$  is represented as shown below.



### Zero pairs

Two identical tiles with opposite signs add up to 0. Such pairs of tiles are called 'zero pairs'.

Example1 Simplify  $3x + 2x - x$



After removing one zero pair, we get  $4x$  as the answer.

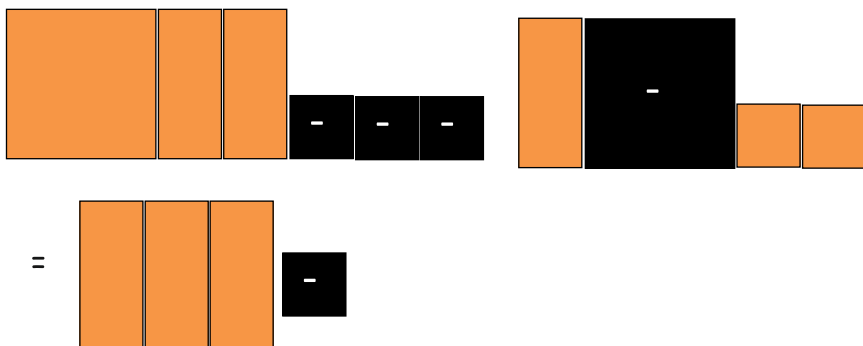
Now find the answers to the questions given in the Activity Sheet.

# Activity 30 ADDITION, SUBTRACTION AND MULTIPLICATION OF EXPRESSIONS using Algebra Tiles.

You may use the same tiles made in Activity 7.

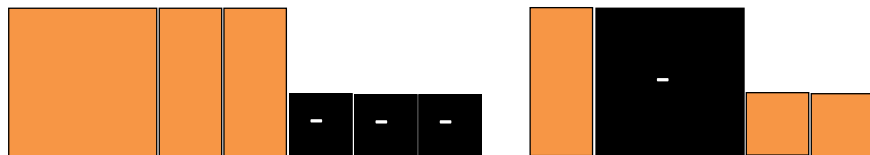
Example 1 Add  $x^2 + 2x - 3$  and  $x - x^2 + 2$

Arrange the tiles as shown below.



After adding, there are 3 zero pairs to be taken off. We are left with  $3x - 1$  which is the answer.

**Subtraction**  $(x^2 + 2x - 3) - (x - x^2 + 2)$



To do subtraction, *flip the tiles* in the 2<sup>nd</sup> expression and then add as shown below.



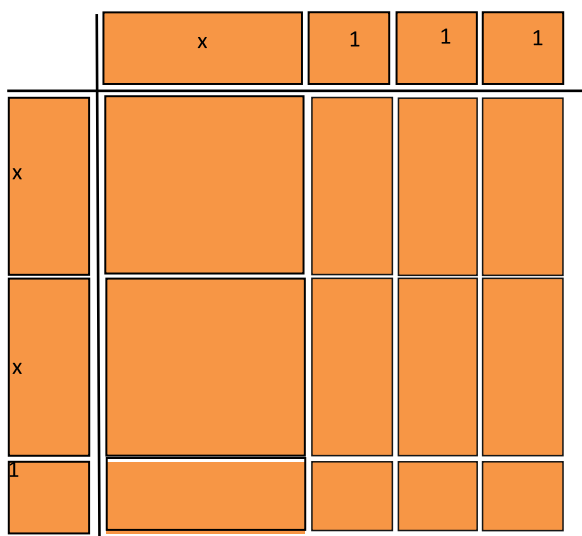
After removing zero pairs we get the answer as  $2x^2 + x - 5$ .

## Multiplication of linear expressions using tiles

Example 1 Multiply  $2x + 1$  by  $x + 3$

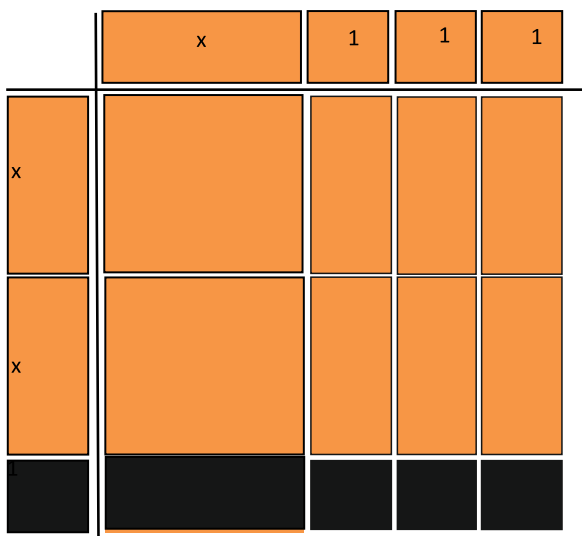
Draw 2 perpendicular lines as shown. Arrange tiles to get  $x + 3$  above the horizontal line and  $2x + 1$  to the left of the vertical line. Fill the space inside with suitable tiles as shown below.

Answer is written down from these 12 tiles. Adding all these, we get  $2x^2 + 7x + 3$  which is the answer. Do not include the tiles above or left of the lines.



Example 2 Multiply  $2x - 1$  by  $x + 3$

Draw 2 perpendicular lines as shown. Arrange tiles to get  $x + 3$  above the horizontal line and  $2x - 1$  to the left of the vertical line. Fill the space inside with suitable tiles as shown below and remove zero pair. Answer is  $x^2 + 5x - 3$



### Activity 31 FACTORIZATION OF ALGEBRAIC EXPRESSIONS OF SECOND DEGREE.

*Materials:* KG card of size 8 X 8 cm (5 cards)

*Procedure*

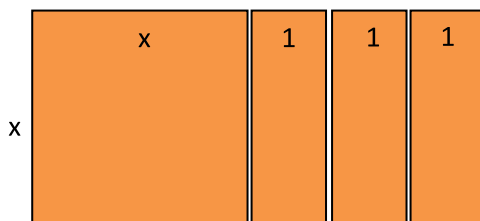
Make 5 sets of Algebra tiles.

You may have to use all the 10 sets of tiles here.

Example 1 Factorize  $x^2 + 3x$

Step 1 Choose one big square and 3 rectangles (positive sides)

Step 2 Arrange all the 4 pieces as a rectangle



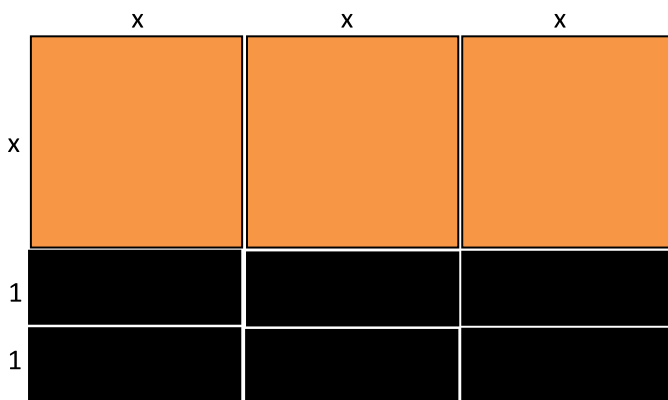
Length of the rectangle =  $x + 1 + 1 + 1 = x + 3$

Breadth of the rectangle =  $x$

Answer =  $x(x + 3)$

Example 2 Factorize  $3x^2 - 6x$

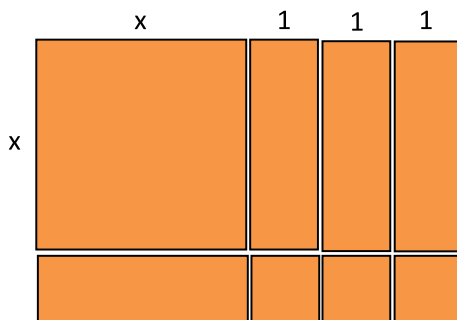
Choose 3 big square tiles and 6 rectangular tiles (negative side) and arrange all the 9 pieces as a rectangle.



Length =  $3x$  and width =  $x - 2$  and hence answer =  $3x(x - 2)$

Example 3 Factorize  $x^2 + 4x + 3$

Choose 1 big tile, 4 rectangles and 3 unit tiles and arrange all the pieces as a rectangle.



$x + 3$  (length) and  $x + 1$  (breadth) are the factors.

## Activity 32 ALGEBRA FROM CALENDAR SHEETS (GAME)

*Materials:* Old calendar sheet

**Game 1** Show any calendar sheet to a friend and ask him or her to choose a  $3 \times 3$  grid and add all the 9 numbers in the grid. As soon as they choose the grid, you have to announce the answer. The secret is that the sum of the numbers in the grid will always be 9 times the middle number. For example,

1	2	3
8	9	10
15	16	17

In the above  $3 \times 3$  grid, sum of all numbers =  $81 = 9 \times 9$

The algebraic proof can be given by taking the first number as  $n$ .

Therefore,  $n + n + 1 + n + 2 + n + 7 + n + 8 + n + 9 + n + 14 + n + 15 + n + 16 = 9n + 72$

$= 9(n + 8)$ , but  $(n + 8)$  is the middle number. Therefore the sum is 9 times the middle number.

**Game 2** Choose any  $4 \times 5$  or  $5 \times 4$  grid. The sum of all the numbers in the grid is

$10 \times$  (first term + last term). In the grid given below, the sum of the numbers is

$10 \times (1 + 26) = 270$ .

The proof for this can be given by taking the first term as  $n$ .

1	2	3	4	5
8	9	10	11	12
15	16	17	18	19
22	23	24	25	26

### Challenge!

In the following number pyramid, what are the values of  $x$  and  $y$ ?

48			
$x+y+4$		$2y+17$	
$x+2$	$y+2$	$y+15$	
$x$	2	$y$	15

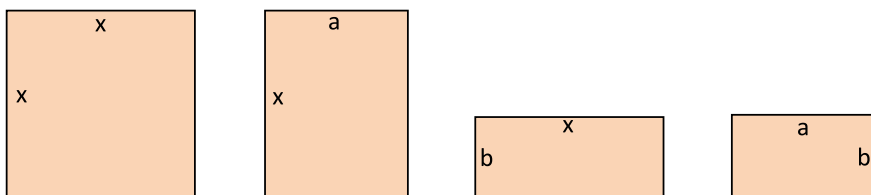
**Activity 33 Proof of identity**  
 $(x + a)(x + b) = x^2 + (a + b)x + ab$   
 using cardboards.

*Materials:* KG card of size about 12 X 10 cm, scissors.

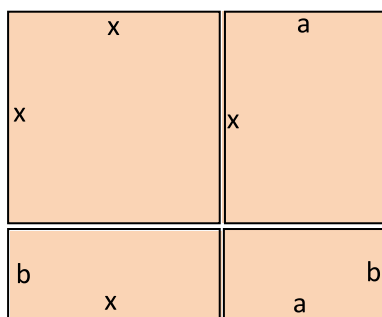
*Procedure*

*Step 1* Cut the cardboard into a square of side 6cm and 3 rectangles of dimensions 6cm X 4cm, 6cm X 2cm and 4cm X 2cm. All the measurements should be very accurate.

*Step 2* Mark the dimensions of all the pieces as shown below.



*Step 3* Arrange the 4 pieces as a big rectangle as shown below.



*Step 4* Area of this rectangle =  $(x + a)(x + b)$  because length is  $x + a$  and breadth is  $x + b$ .

This area is equal to sum of areas of the 4 pieces =  $(x^2) + (ax) + (bx) + (ab) = x^2 + (a + b)x + ab$

Stick the pieces on your Activity sheet.

### Activity 34 Verification of the identity

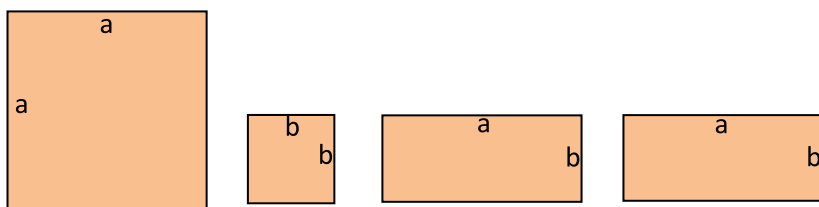
$$(a + b)^2 = a^2 + 2ab + b^2$$

*Materials:* KG card of size about 10cm X 10cm, scissors

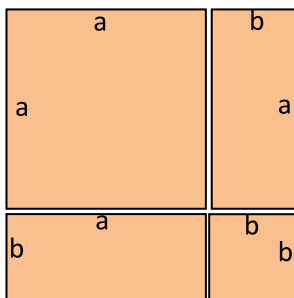
#### *Procedure*

*Step 1* Cut the sheet into 2 squares of sides 5cm and 3cm and 2 rectangles of dimensions 5cm X 3cm and 5cm X 3cm as shown below. You have to cut neatly and exactly to measurements.

*Step 2* Mark all the dimensions as shown below



*Step 3* Arrange the 4 pieces as a big square as shown below.



Area of the whole square =  $(a + b)^2$  because each side is  $a + b$ .  
This area is equal to

Area of square of side  $a$  + area of rectangle of length  $a$  and breadth  $b$  + area of rectangle of length  $a$  and breadth  $b$  +

area of the square of side b

$$= a^2 + ab + ab + b^2$$

$$\text{Therefore, } (a + b)^2 = a^2 + 2ab + b^2$$

Now stick the pieces with proper labeling in your Activity sheet.

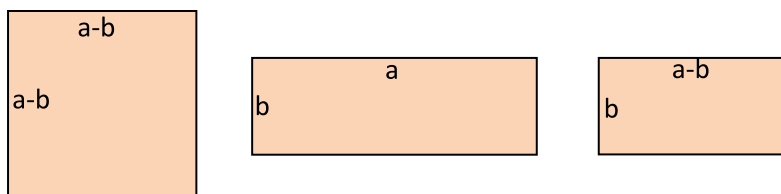
### Activity 35 Verification of $(a - b)^2 = a^2 - 2ab + b^2$

*Materials* KG card of size about 10cm X 10cm, scissors

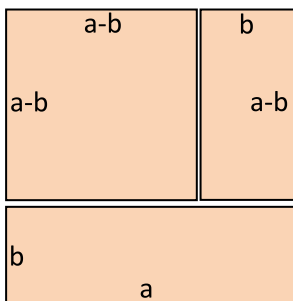
*Procedure*

*Step 1* From the given card, cut out 3 pieces: a square of side 5cm, a rectangle of dimensions 5cm X 3cm and another rectangle of size 8cm X 3cm. Measure accurately and then cut.

*Step 2* Mark the dimensions of the 3 pieces as shown below.



*Step 3* Arrange the 3 pieces as shown below



Area of inner square =  $(a - b)^2$

This area = area of whole square - area of small rectangle - area of big rectangle

That is,  $(a - b)^2 = a^2 - (a - b)b - ab = a^2 - ab + b^2 - ab = a^2 - 2ab + b^2$

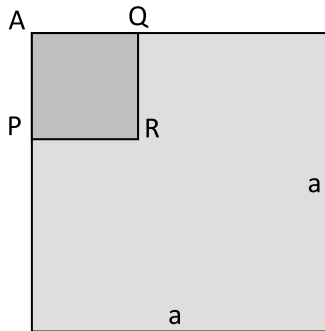
On the Activity sheet, stick the cards as shown above with all the markings and write the proof.

### Activity 36 Verification of $a^2 - b^2 = (a + b)(a - b)$

**Materials:** KG card of size 10 X 10 cm (2 pieces), scissors

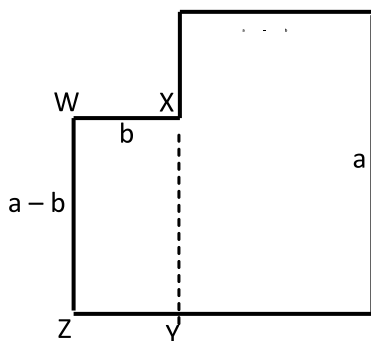
**Procedure**

**Step 1** On the square card, mark points P and Q as shown below such that  $AP = AQ$ . Complete the square APRQ. Mark the side of the full square as 'a' and that of the small square as 'b'.

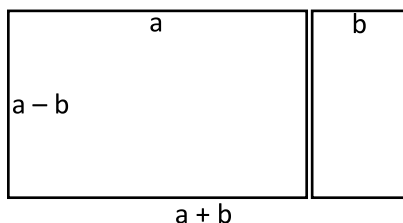


**Step 2** Take a pair of scissors and cut along PR and RQ. Now you have removed a square of area  $b^2$  from a square of area  $a^2$ . In other words, you subtracted  $b^2$  from  $a^2$ .

Therefore the area of the remaining figure as shown below, must be  $a^2 - b^2$ .



On the Activity sheet, stick the above piece in the first box with all the markings. To find the area of this shape, cut along the dotted line  $XY$  and take out a rectangle  $XYZW$ . Place the rectangle as shown below so that you get a bigger rectangle of dimensions  $a + b$  and  $a - b$ . Its area is  $(a + b)(a - b)$ .



Stick the above rectangle in the Activity sheet in the 2<sup>nd</sup> box.

### Activity 37 Verification of identity

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

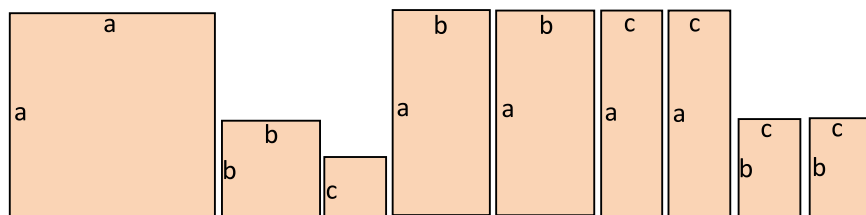
**Materials:** KG card of size about 15 X 15 cm and scissors

**Procedure**

**Step 1** Cut the card into 9 pieces :

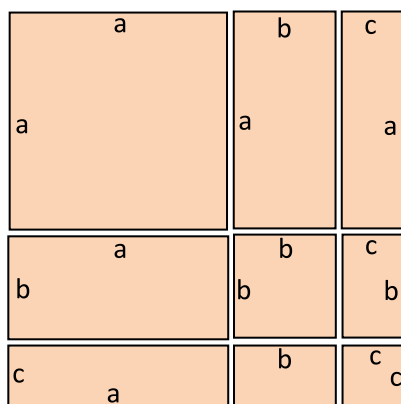
- ✓ A square of side 6 cm
- ✓ A square of side 4 cm
- ✓ A square of side 2 cm
- ✓ 2 rectangles of dimensions 6cm X 4cm each
- ✓ 2 rectangles of dimensions 4cm X 2 cm each

2 rectangles of dimensions 6cm X 2 cm each



**Step 2** Mark the dimensions on all the pieces as shown.

**Step 3** Arrange all the 9 pieces as a square as shown below.



Area of the whole square =  $(a + b + c)^2$  because the length of each side is  $a + b + c$ .

This area = area of 3 inner squares + area of 6 rectangles

$$= a^2 + b^2 + c^2 + ab + ab + bc + bc + ac + ac$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

### Activity 38 Proof of identity

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

*Materials:* 27 small cubes of the same size.

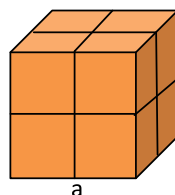
*Procedure*

If the identity is interpreted geometrically,  $a^3$  is the volume of a cube of side  $a$ ,

$b^3$  is the volume of a cube of side  $b$ ,  $a^2b$  is the volume of a cuboid of dimensions

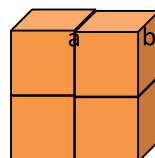
$a \times a \times b$  and  $ab^2$  is the volume of a cuboid of dimensions  $a \times b \times b$ . Thus you have to get 2 cubes and 6 cuboids. It is possible to make all these solids using the 27 cubes you have with you as follows.

*Step 1* Take 8 small cubes and make a cube of dimensions  $a \times a \times a$



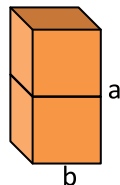
*Step 2* Take 4 cubes and make a cuboid of dimensions  $a \times a \times b$

You have to make 3 copies of this cuboid.

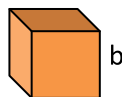


*Step 3* Take 2 cubes and make a cuboid of dimensions  $a \times b \times b$

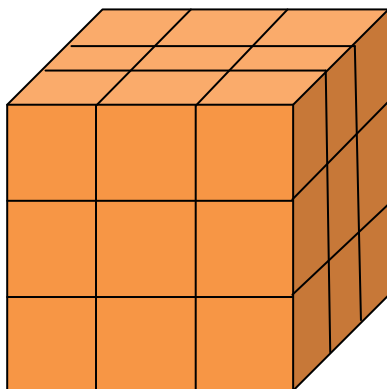
You have to make 3 copies of this cuboid



*Step 4* One cube will be of dimensions  $b \times b \times b$



*Step 5* Assemble all the 8 solids and get a cube of side  $a + b$  :



Volume of the whole cube =  $(a + b)^3$

Sum of volumes of the 8 solids =  $a^3 + 3a^2b + 3ab^2 + b^3$

### Activity 39 Verification of the identity

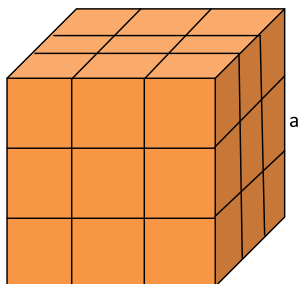
$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

**Materials:** 27 small cubes of the same size

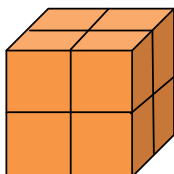
**Procedure**

The same set of 2 cubes and 6 cuboids made in Activity 17 have to be made here.

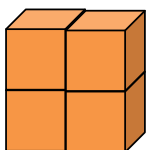
But *dimensions have to be changed as follows:*



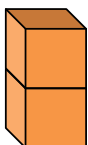
Assume that this full cube has dimensions  $a \times a \times a$  and the side of *each small cube* as ' $b$ '.



This cube has side  $(a - b)$  and hence volume =  $(a - b)^3$



This cuboid has dimensions  $(a - b) \times (a - b) \times b$  and hence volume =  $(a - b)(a - b)b = (a^2 - 2ab + b^2)b = a^2b - 2ab^2 + b^3$



This cuboid has dimensions  $(a - b) \times b \times b$  and hence its volume =  $(a - b) \times b \times b = ab^2 - b^3$



This cube has side  $b$  and hence its volume =  $b^3$

Cube of volume  $(a - b)^3 = \text{cube of volume } a^3 - 3 \times \text{cuboid of volume } (a^2b - 2ab^2 + b^3)$

$- 3 \times \text{cuboid of volume } (ab^2 - b^3) - \text{cube of volume } b^3$

$$\begin{aligned} \text{That is, } (a - b)^3 &= a^3 - 3a^2b + 6ab^2 - 3b^3 - 3ab^2 + 3b^3 - b^3 \\ &= a^3 - 3a^2b + 3ab^2 - b^3 \end{aligned}$$

#### Activity 40 Verification of the identity

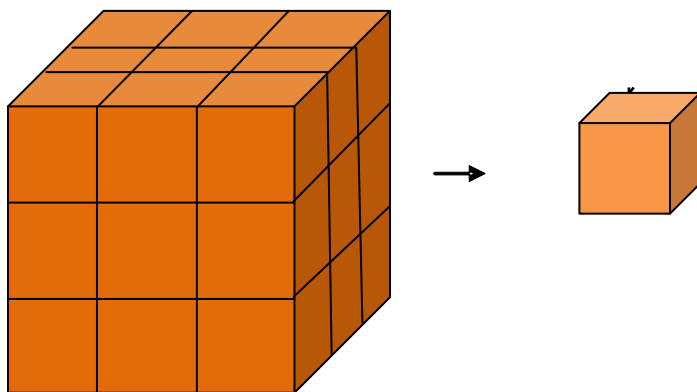
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

*Materials:* 27 identical cubes.

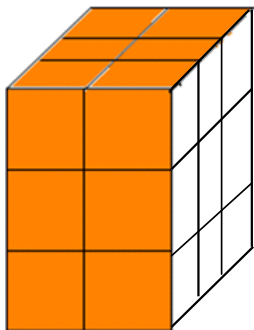
#### *Procedure*

Labeling the dimensions have to be different from the earlier activities. Let us first take the whole cube made up of 27 small ones. Assume each side of this cube as 'a' and each side of a small cube as 'b'.

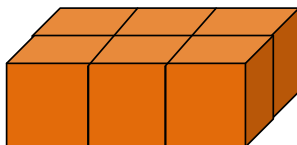
To get  $a^3 - b^3$ , we have to take away a small cube from the whole cube leaving 26 small cubes behind.



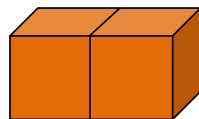
From the remaining 26 cubes, you have to make 3 cuboids of different dimensions as shown below.



$(a - b) \times a \times a$   
(join 18 small cubes)



$(a - b) \times a \times b$   
(join 6 small cubes)



$(a - b) \times b \times b$   
(join 2 small cubes)

$$\text{Therefore } a^3 - b^3 = (a - b) a^2 + (a - b) ab + (a - b) b^2 = (a - b)(a^2 + ab + b^2)$$

### Activity 41 EXPONENTS AND POWERS

We make a **NOMOGRAPH** to illustrate laws of exponents

**Materials :** graph sheet

- ✓ Draw three equidistant parallel lines AB, CD and EF as shown in the figure below. (Do not write letters A, B etc. It is only for your reference)
- ✓ Mark  $2^0$  at the mid point of the first line AB, positive powers of 2 above and negative powers below at intervals of 2 cm. Repeat this procedure on the 3<sup>rd</sup> line EF.
- ✓ Mark  $2^0$  as the mid point of the middle line CD . Mark the positive powers of 2 above and negative powers below at 1 cm intervals.

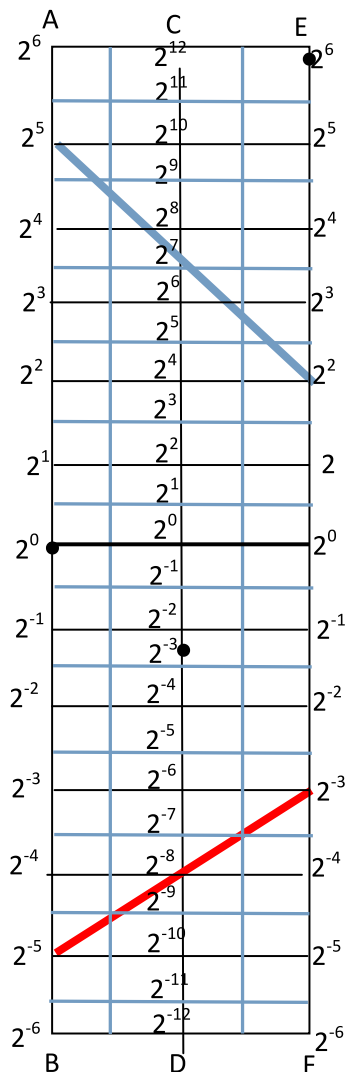
Your nomograph is ready!

## How to use?

- ✓ To find the product of  $2^5$  and  $2^2$  take a thin stick or a thread and connect the point at  $2^5$  on AB and the point  $2^2$  on EF and see where it meets the middle line CD. This point gives the answer  $2^7$ .
- ✓ To find  $2^{-5} \times 2^{-3}$  connect  $2^{-5}$  on AB with  $2^{-3}$  and see where it cuts CD. The answer is  $2^{-8}$ .

To verify *Product Law of exponents*, replace '2' by 'a' in the nomograph. You will get powers of a. Then it can easily be seen that  $a^m \times a^n = a^{m+n}$ .

Furthermore, suppose we want  $2^3 \div 2^2$ , connect with a stick or thread, the point  $2^3$  in the **middle** line with  $2^2$  in the **first** line, answer  $2^1$  will appear on the **third** line (shown by dotted line). In the nomograph having powers of 'a' this property becomes *Quotient Law*, namely  $a^m \div a^n = a^{m-n}$ .



## Activity 42 In this activity you will learn to SOLVE LINEAR EQUATIONS USING CARDBOARDS.

**Materials:** 4 circular KG cards of diameter 3 cm; 10 rectangular cards of size 5 X 3 cm

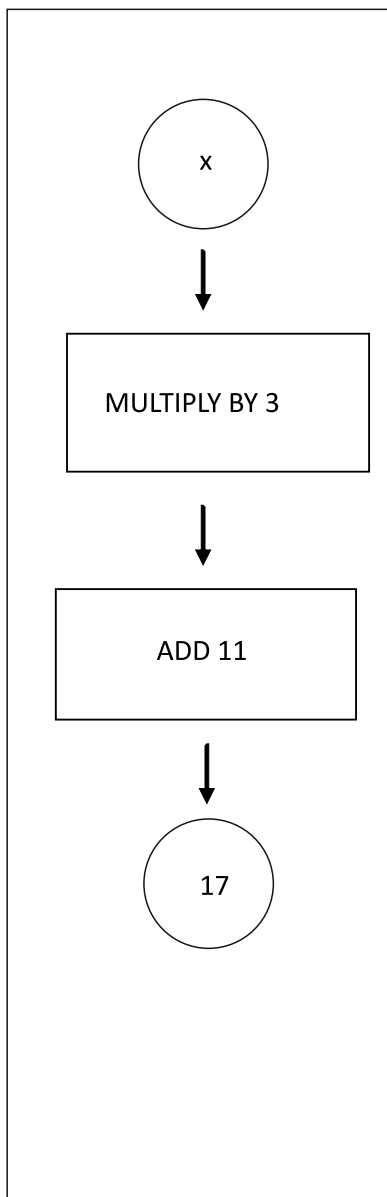
**Illustration :** Let us take the following problem.

*I took a number, multiplied it by 3 and added 11. If my answer is 17 what was the number I took?*

Let  $x$  be the required number.

Take a circular card and write  $x$  in the middle, leaving the other side blank. On a rectangular card write MULTIPLY BY 3 and on the opposite side, write DIVIDE BY 3. On another rectangular card write ADD 11 and on the opposite side write SUBTRACT 11. On another circular card write 17, leaving the other side blank.

The given question can be represented diagrammatically as shown.



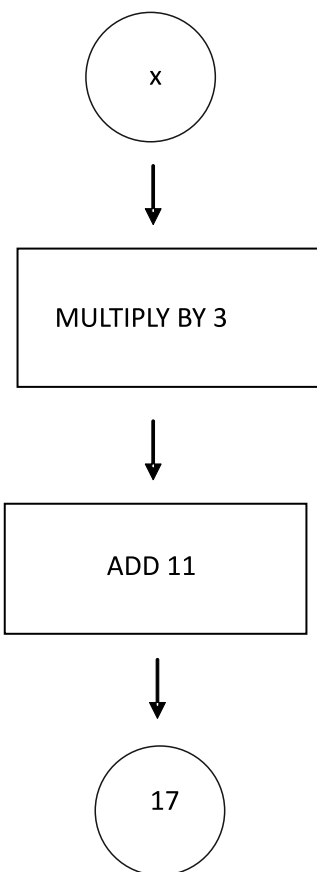
To find the answer you have to reverse all the steps.

Start from the bottom.

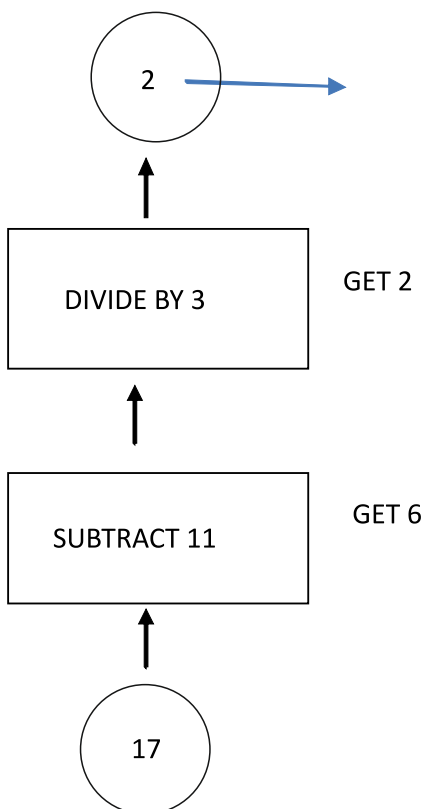
Leave the last card having 17 on it as it is.

FLIP ALL THE OTHER CARDS AS SHOWN BELOW.

Problem



Solution



## Second Method using cards

### Procedure

Cut 6 rectangular cards as shown below along the dotted lines into 12 rectangles and 12 squares. Mark the dimensions of each rectangle as  $x$  and  $1$  and the side of each square as  $1$ . Write a big plus sign on one side and a minus sign on the other side

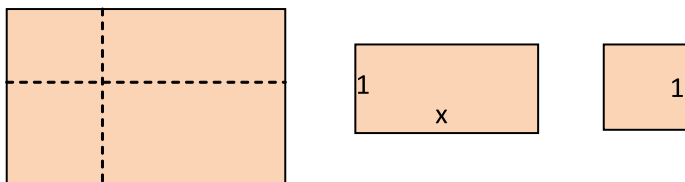
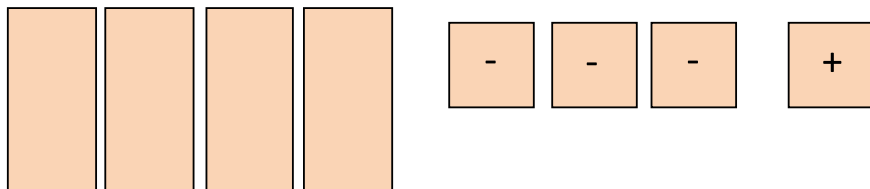


Illustration Solve  $4x - 3 = 1$

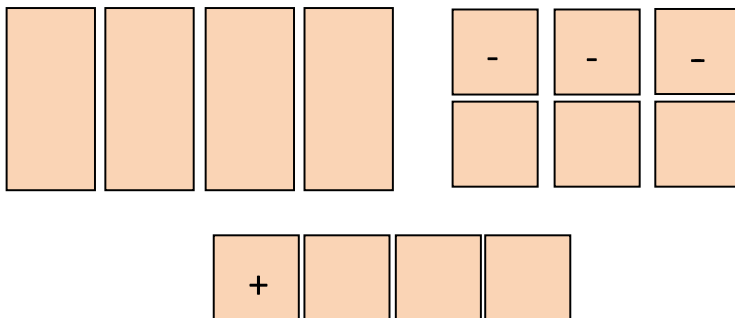
Choose 4 rectangular cards, 3 unit cards (negative side up) and 1 unit card (positive side up)

Write the equation using cards as shown.

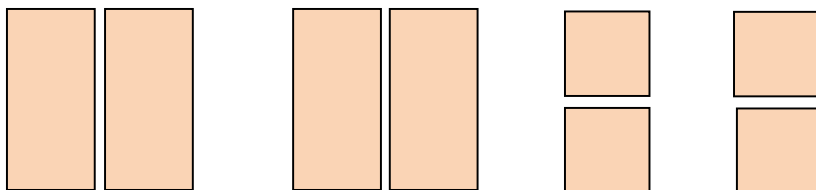


Since we want only  $x$ , the 3 unit cards have to be removed from the left side.

This can be done by adding 3 unit cards (positive side) to both sides. Thus we have,



Remove all the zero pairs and arrange the tiles as shown below.



Taking corresponding groups, we can get  $x = 1$  as the solution.

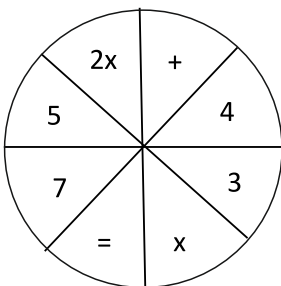
### Activity 43 EQUATION WHEELS

*This activity is a practice activity for framing simple equations in algebra.*

**Materials:** Plain paper

'Equation wheel' is a mix of constants and variables arranged in a circular way. You have to frame an equation using some or all of these, the solution for which is also in the same wheel. Only the symbols given in the wheel must be used.

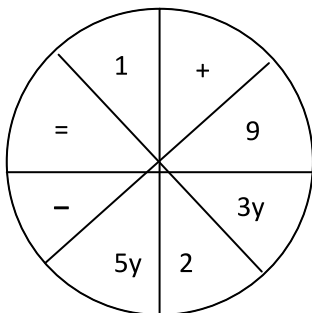
Illustration



One equation is  $x + 3 = 7$  solution being 4.

You have to find some more equations from the same wheel.

How many equations can you find from the wheel below?



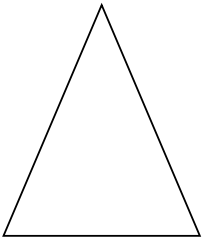
# GEOMETRY

## Activity 44 DIFFERENT TYPES OF TRIANGLES I

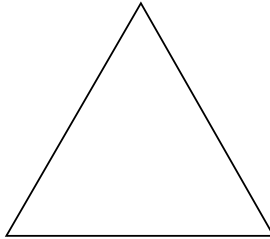
As an introduction to triangles, first do the coloring activity given in the Activity Sheet and then go to the following activity.

**Materials:** 5 thin straws and pipe cleaners, cellotape, scissors

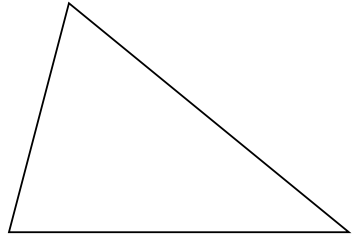
*Procedure*



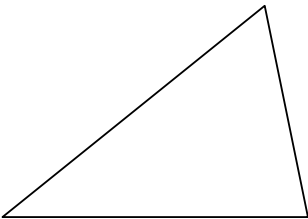
ISOSCELES



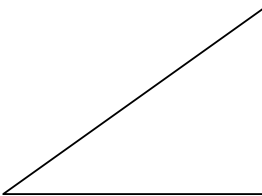
EQUILATERAL



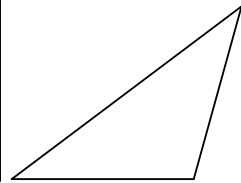
SCALENE



ACUTE



RIGHT



OBTUSE

Insert the pipe cleaners into the straws to make them stiff. Make 5 types of triangles as accurately as possible. *If pipe cleaners are longer than the straws, the extra length may be inserted into the straws at the end and strengthen the triangles.* After making the models stick them in the Activity Sheet and label them. Then the table in the AS has to be completed.

#### **Activity 45   DIFFERENT TYPES OF TRIANGLES II**

*Materials:* Color pencils

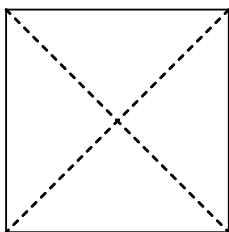
*Procedure*

Take the Activity Sheet and color each type of triangle in a different color and then complete the table printed in the sheet.

## Activity 46 MAKING TRIANGULAR SHAPES

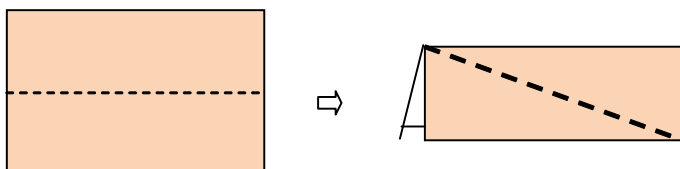
Materials: Craft paper (10 sheets in different colors), scissors.

**(a) Right triangle.** Fold a square sheet of paper along the diagonals, make creases and unfold. Cut the sheet into 4 triangular pieces. Each is a right triangle.

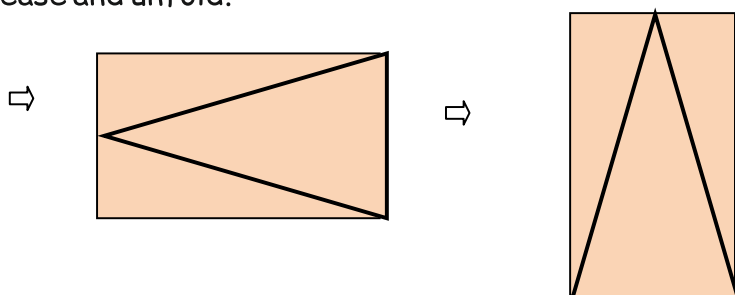


**(a) Isosceles triangle.**

**Step 1** Start with a rectangular sheet of paper. Fold into half lengthwise.

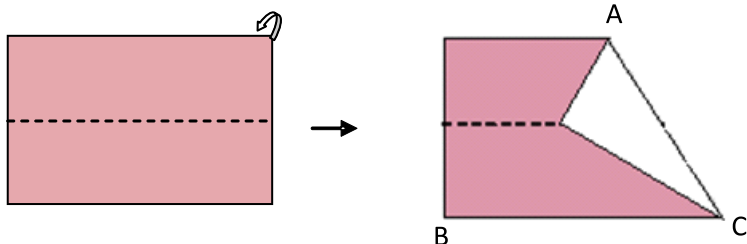


**Step 2** Fold the rectangular piece along the diagonal, make a crease and unfold.



Cut and get an isosceles triangle.

(c) **Equilateral triangle.** **Step 1** Start with a rectangular sheet of paper. Fold into half lengthwise, make a crease along the dotted line and unfold.

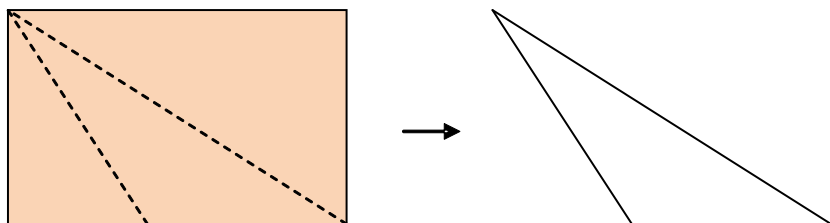


**Step 2** Bring the top right corner A onto the middle crease as shown above.

**Step 3** Cut along AB and AC and get an equilateral triangle.

(d) **Obtuse triangle.**

Start with a rectangular sheet of paper. Mark any point X on the base of the sheet. Cut along the dotted lines, you will get an obtuse triangle.



### Activity 47 INEQUALITY PROPERTY OF TRIANGLES

Materials: Thin sticks of sizes 3cm, 4cm, 5cm, 6cm and 10cm- 4 each in a packet

#### Procedure

Choose any 3 sticks of different lengths at random and make triangles. You should observe that triangles cannot be made with some sets of 3 sticks. Prepare a table in the Activity sheet. From the table it is possible to conclude that a triangle can be made only when sum of 2 sides is greater than the third.

#### Questions for you

The lengths of sides and measures of angles of some triangles are given below. Match the items in the first column with those in the second by drawing ARROWS.

DO NOT DRAW THE TRIANGLES.

ABC with  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$ ,  $AC = 4\text{cm}$

DEF with  $DE = 7\text{ cm}$ ,  $EF = 5\text{ cm}$ ,  $DF = 7\text{ cm}$  Isosceles

XYZ with  $XY = 12\text{ cm}$ ,  $YZ = 12\text{ cm}$ ,  $XZ = 12\text{ cm}$  scalene

PQR with angle  $P = 25^\circ$ ,  $Q = 140^\circ$ ,  $R = 15^\circ$  acute

LMN with angle  $L = 28^\circ$ ,  $M = 90^\circ$ ,  $N = 62^\circ$  equilateral

JKL with angle  $J = 60^\circ$ ,  $K = 60^\circ$ ,  $L = 60^\circ$  obtuse

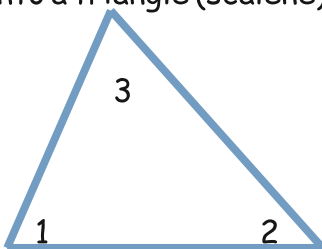
FGH with  $FG = 8\text{ cm}$ , angle  $F = 45^\circ$ , angle  $G = 45^\circ$  right

BMN with angle  $B = 40^\circ$ , angle  $M = 60^\circ$ , angle  $N = 80^\circ$

### Activity 48 ANGLE SUM PROPERTY OF TRIANGLES.

**Materials:** 3 thin straws; 3 pipe cleaners, scissors, cellotape

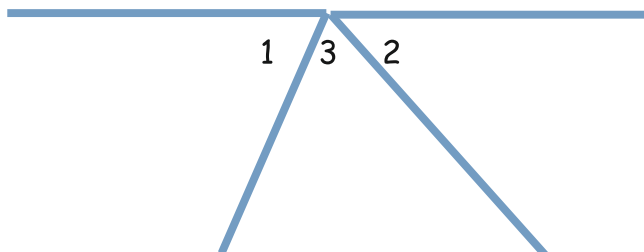
**Procedure** 2 straws should be joined by inserting one into the other to get a total length of about 20 cm and then made stiff by inserting pipe cleaners. The ends should be joined before bending it into a triangle (scalene).



Next, make 2 angles, using straws, equal to the base angles of the triangle made. See figure below.



When you place the 3 pieces as shown below, a straight angle is formed at the top vertex, thus proving the property.

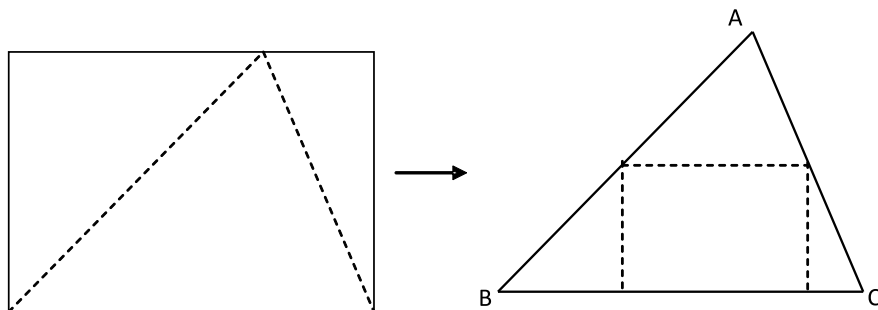


In the Activity Sheet, using cellotapes affix the triangle and the angles as shown in the above figure with angles marked as 1, 2, 3. Write your conclusion below the box.

## Activity 49 ANGLE SUM PROPERTY BY PAPERFOLDING

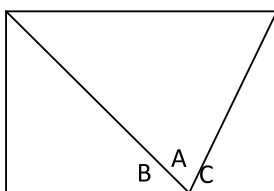
**Materials:** Craft paper (10 X 6 cm), scissors

**Procedure**  
**Step 1** Cut out a scalene triangle from the given sheet.



**Step 2** Fold the triangular sheet carefully about the horizontal dotted line such that the vertex  $A$  just touches the base.

**Step 3** Fold the corners  $B$  and  $C$  about the vertical dotted lines so that  $B$  and  $C$  fall next to  $A$  on the base of the triangle.



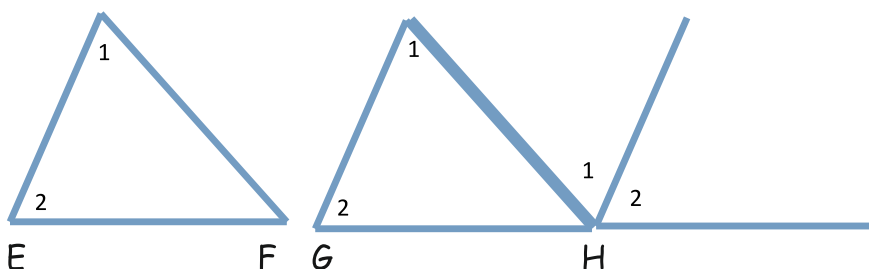
Stick the model in the Activity sheet as instructed in it.

## Activity 50 EXTERIOR ANGLE PROPERTY OF TRIANGLE

**Materials:** 3 thin straws, pipe cleaners, scissors, cellotape

### Procedure

This is almost same as Activity 48. You have to make a scalene triangle and 2 interior opposite angles with straws and stick them together appropriately on the Activity Sheet.

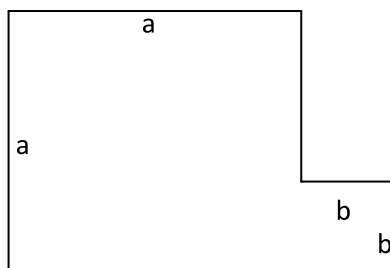


## Activity 51 PYTHAGORAS' THEOREM

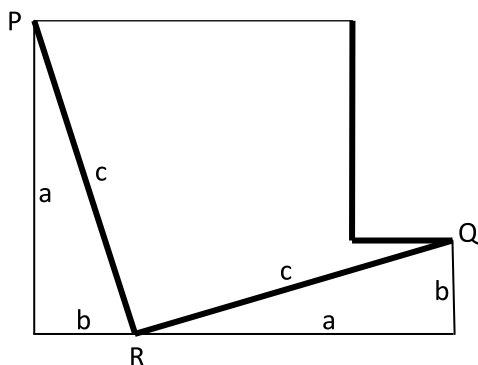
**Materials:** Cardboard of size 15 X 12cm, ruler, scissors

**Procedure** Cut the card as shown below and write the dimensions.

$$\text{Total Area} = a^2 + b^2$$



Mark a point R on the base at a distance 'b' as shown below and connect R to P and Q using a ruler.

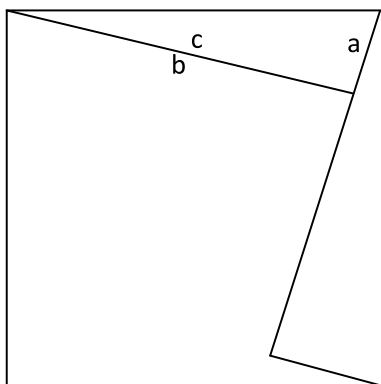


With scissors cut carefully along the thick lines. You will get 2 right triangles and a third shape (irregular pentagon)

Your model is now ready.

Arrange the pieces to get a big square as shown below. Area of this square is  $c^2$ .

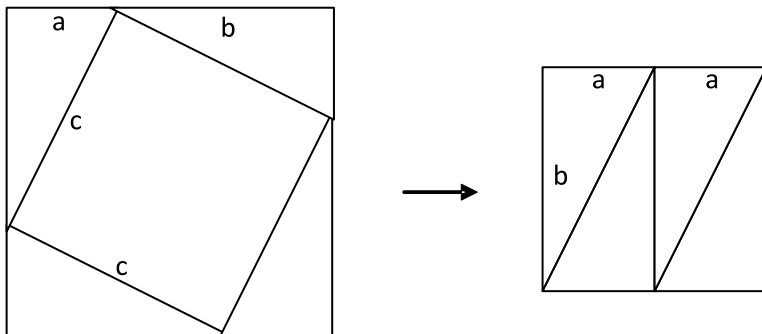
Hence,  $a^2 + b^2 = c^2$  which verifies the theorem for any one of the triangles in the model.



## (2<sup>nd</sup> proof)

**Materials:** A square card of size 10 X 10 cm and scissors

**Procedure:** Mark points at the edges of the sheet as shown below. You will get a square and 4 congruent right triangles surrounding it. Mark the dimensions of the triangles as  $a$ ,  $b$ ,  $c$  and also the side of the square as  $c$ .



With scissors, cut the card into 4 triangles. A square (area of square =  $c^2$ ) will be left.

Place the 4 triangles as shown in the form of a rectangle of length  $b$  and breadth  $2a$

(fig 2) so that its area is  $2ab$ .

Now, area of the whole sheet =  $(a + b)^2$

Therefore  $c^2 = (a + b)^2 - 2ab = a^2 + 2ab + b^2 - 2ab = a^2 + b^2$ .

Hence  $c^2 = a^2 + b^2$  which is Pythagoras' theorem.

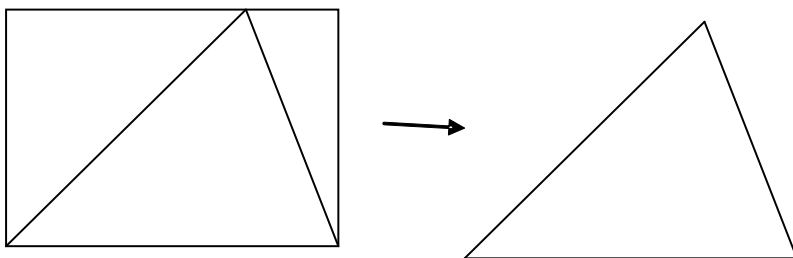
## Activity 52 TRIANGLE CENTRES BY PAPER FOLDING

*Materials:* Craft paper size 10 X 8 cm (4 sheets), scissors, stapler, geometry box

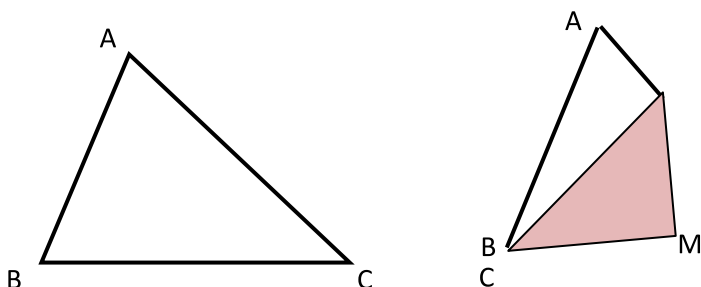
- **Centroid (point of concurrence of medians)**

### Procedure

*Step1* Cut a scalene triangle from all the sheets as shown below.

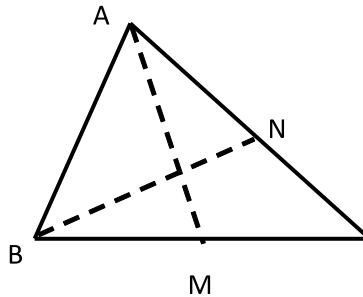


*Step 2* Bring the corner  $C$  onto  $B$  without folding the sheet completely and 'pinch' the sheet in the middle. The pinch mark made will be the midpoint  $M$  of  $BC$ .



*Step 3* Now hold the sheet with your left hand and fold it along  $AM$  making a crease. Unfold. This crease  $AM$  will be one median.

*Step 4* Turn the triangle round and find the second median  $BN$  in the same manner.



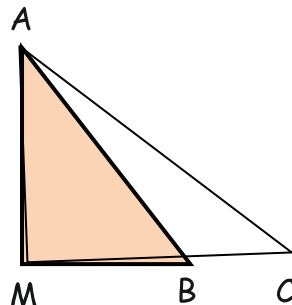
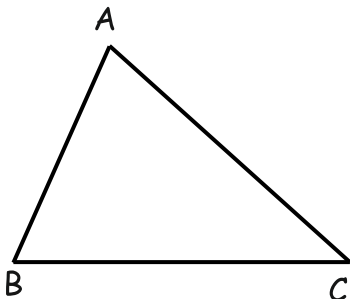
The point of intersection of these medians will be the **Centroid**.

Furthermore, by finding the third median it can be verified that the three medians of a triangle are concurrent.

- **Orthocentre** (point of concurrence of altitudes)

Start with a scalene triangle  $ABC$ .

*Step 1* Lift the corner  $B$  and fold it onto the base  $BC$  so that  $B$  falls on the base and a right triangle is formed.



Make a crease along  $AM$ . This will be one altitude of the triangle.

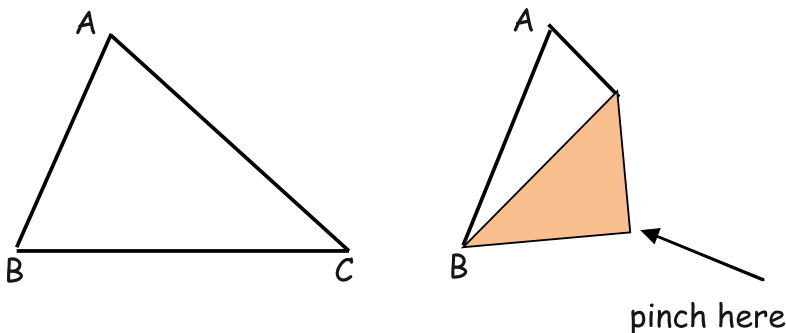
*Step 2* Turn the triangle round and make a crease for the second altitude. The point of intersection of the two altitudes will be the orthocenter.

Furthermore, by making a crease for the third altitude, verify that the altitudes of a triangle are concurrent.

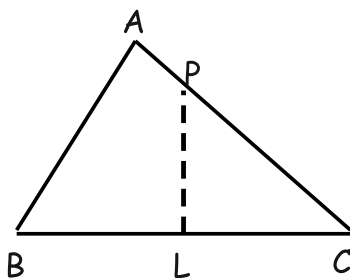
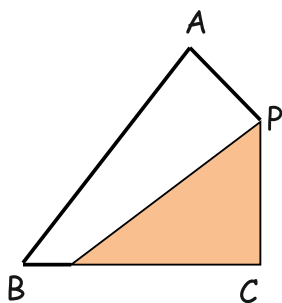
- **Circum center** (the point of concurrence of perpendicular bisectors of sides).

Start with a scalene triangle.

*Step 1* To find the midpoint of a side, bring the corner  $C$  onto the corner  $B$ , without folding it completely and pinch the sheet in the middle to make a mark. This mark will be the midpoint of the base  $BC$ .



*Step 2* Bring vertex  $C$  onto base  $BC$  so that a right triangle is formed and  $C$  falls on  $BC$ . Make a crease and unfold.



The crease PL is the perpendicular bisector of BC.

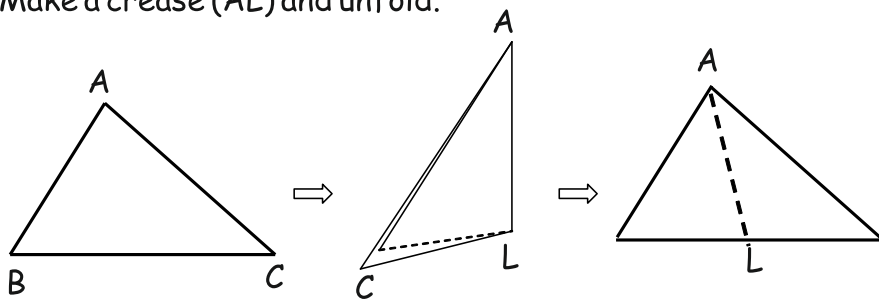
*Step 3* Turn the triangle round and fold another perpendicular bisector. Intersection gives the circum center.

Concurrency of the three perpendicular bisectors can also be verified.

- **In-center** (point of concurrence of the angular bisectors of the triangle)

Start with a scalene triangle.

*Step 1* Fold side AC to the left so that AC falls along AB. Make a crease (AL) and unfold.



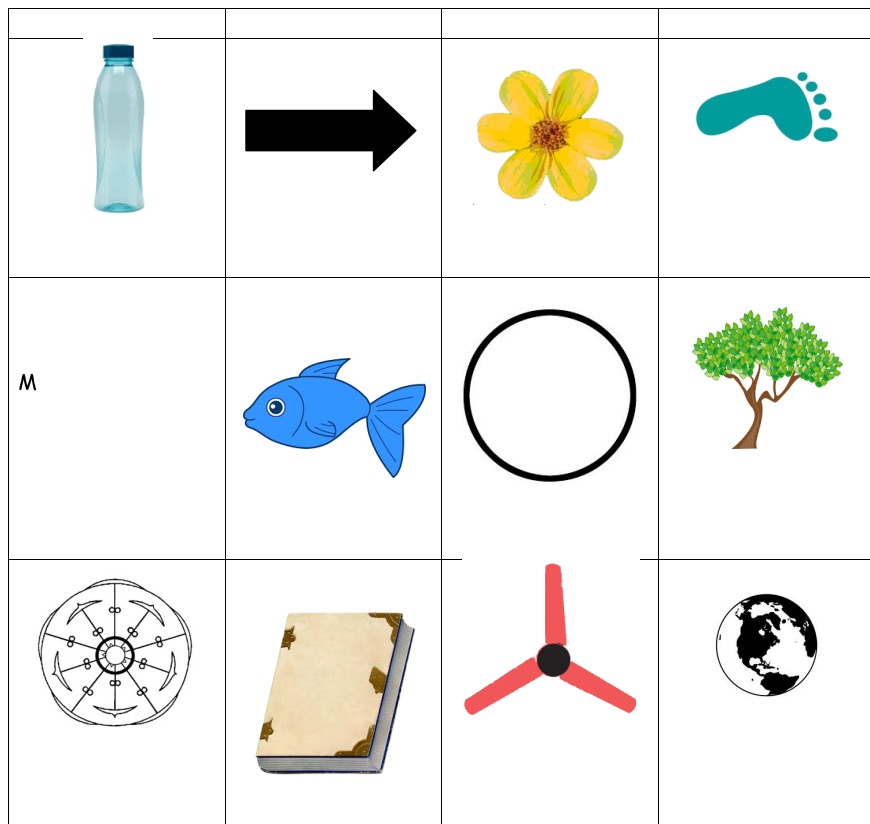
*Step 2* Fold one other angular bisector and get the in-center as their intersection.

Concurrency of angular bisectors of a triangle can also be verified.

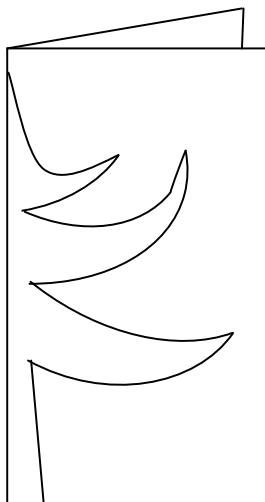
## Activity 53 SYMMETRY

Materials: 12 Cutouts; A4 sheets (2)

(A) In the Activity sheet the following 12 pictures are printed. Cut out all the pictures separately and identify all the symmetric objects by folding into half.



(B) Fold an A4 sheet lengthwise into half and make a crease. *At the folded edge*, draw only the vertical half of a Christmas tree and with scissors cut both layers along the boundary of the tree drawn, keeping the sheet folded all the time.



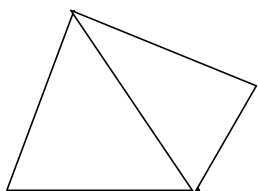
The full tree appears when the sheet is unfolded. You can try other shapes such as a mask, bird, cartoon character etc.

## Activity 54 SUM OF INTERIOR ANGLES OF A POLYGON

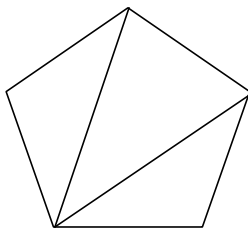
**Materials:** KG card cutouts of 2 yellow, 3 pink and 4 green triangles (scalene) of different sizes and glue.

**Procedure:**

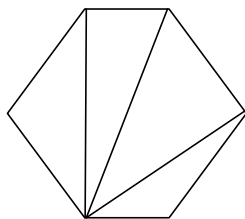
Make a quadrilateral with 2 triangular cutouts of same color, a pentagon (5 sides) with 3 triangles of same color and a hexagon (6 sides) with 4 triangles. You will notice that all polygons are made up of triangles. Sample polygons are shown below.



(quadrilateral)



(pentagon)



(hexagon)

Stick all these in your Activity Sheet in the space provided.

We know that the sum of all the 3 angles of a triangle =  $180^\circ$

$\therefore$  Sum of all the angles of a quadrilateral =  $2 \times 180^\circ = 360^\circ$ , since there are 2 triangles.

Sum of all the angles of a pentagon =  $3 \times 180^\circ = 540^\circ$ , since there are 3 triangles.

Sum of all the angles of a hexagon =  $4 \times 180^\circ = 720^\circ$ , since there are 4 triangles.

In general, sum of all the angles of any polygon =  $(n - 2) \times 180^\circ$  since there are  $(n - 2)$  triangles.

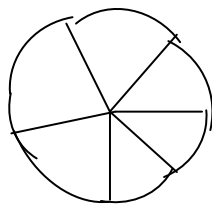
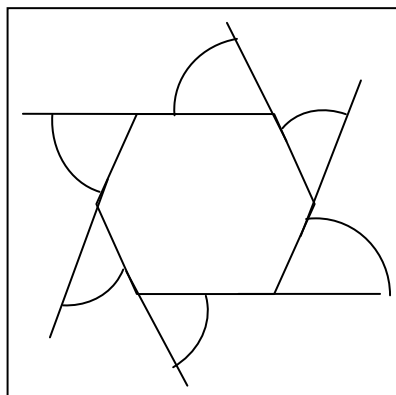
## Activity 55 SUM OF EXTERIOR ANGLES OF A POLYGON

*Materials:* KG card of size 15 X 15 cm and scissors.

*Procedure*

On the card, draw a pentagon or hexagon in the middle as shown below.

Extend all the sides on one side, mark all the exterior angles and color the angles.



Now place all the angles with a common vertex and stick them together in the Activity sheet. The angles together form a complete angle =  $360^\circ$ .

Therefore, the sum of all the exterior angles of any polygon is  $360^\circ$ .

## Activity 56

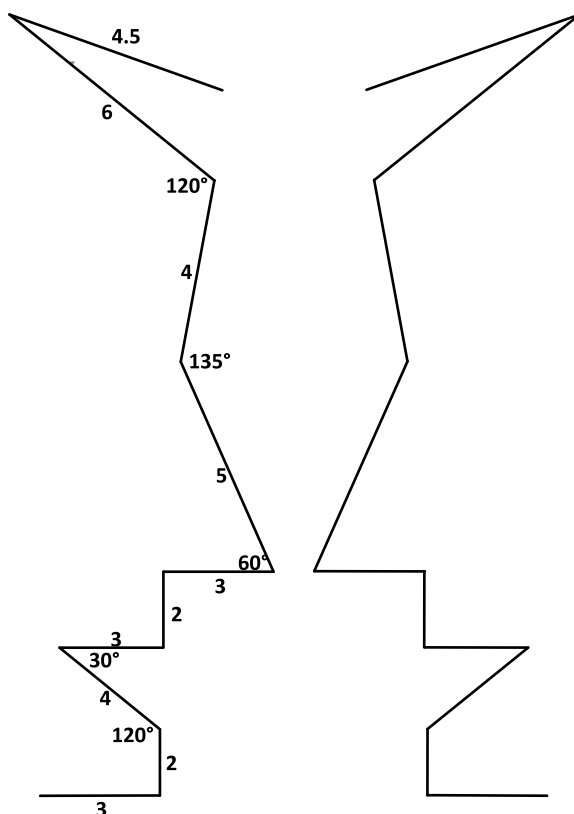
## CONSTRUCTIONS 1(to practice the use of geometrical instruments)

*Materials:* A4 size cardboard, ruler and protractor

### *Procedure*

Use a ruler and protractor and construct accurately the left figure given below. **All the lengths are in cm.**

Now cut the card carefully along the boundary and use it as a template and draw the right part on another card. When you place the 2 pieces as shown, do you see a flower vase or two faces? Stick/staple the model on an A4 sheet.



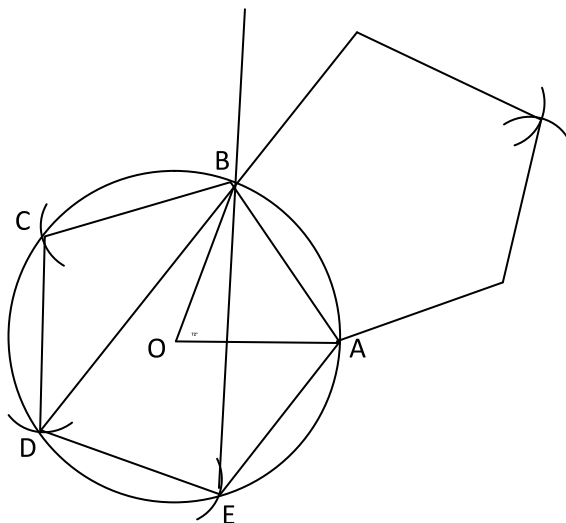
## Activity 57 CONSTRUCTIONS (2)

This construction demonstrates how a construction using protractor and compass culminates in a 3D shape (dodecahedron)

*Materials:* 2 cardboards of size 30 X 30 cm, protractor, ruler, compass and scissors.

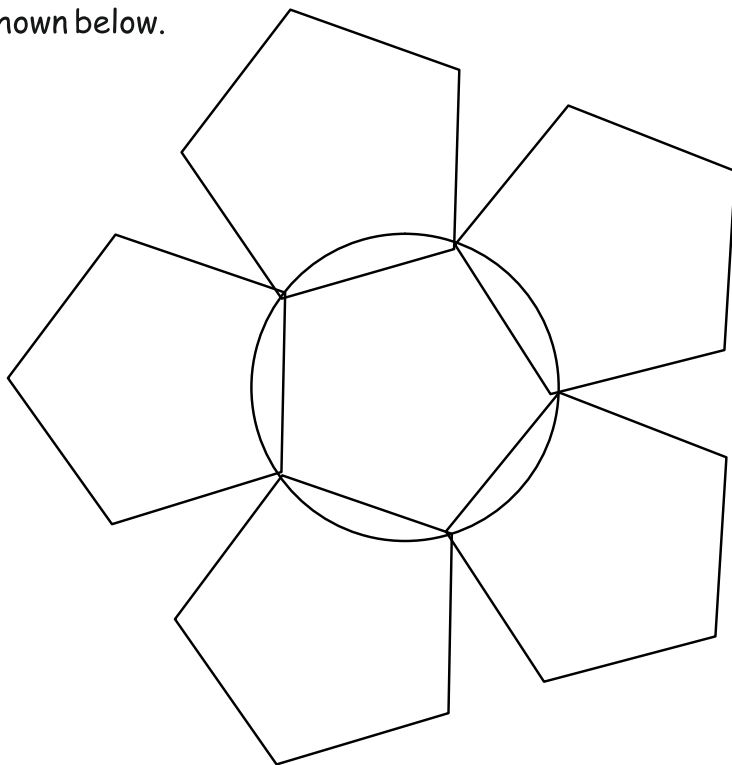
### *Procedure*

*Step 1* At the center of the card draw a circle of radius 4 cm. Draw a radius  $OA$  and make an angle  $AOB = 72^\circ$  at the center (each exterior angle of a regular pentagon is  $360/5 = 72^\circ$ ) using a protractor. Join  $AB$ . With  $B$  as center and radius  $AB$  draw an arc to intersect the circle at  $C$ ; With  $C$  as center and same radius, draw another arc to cut the circle at  $D$ . Continue this till you get a regular pentagon  $ABCDE$ .



*Step 2* In this step we construct 5 regular pentagons on AB, BC, CD, DE and AE in order to get a 'net' for making a bowl. Join DB and EB and extend them as shown and make the extensions equal to AB. Similarly make all the extensions outside the circle equal to the side of the pentagon. Finally use compass and complete all the 5 pentagons outside the circle. One such pentagon is shown above.

*Step 3* Cut carefully along the boundary and get the net shown below.



Make a *fruit bowl* by folding up the model properly and joining the edges with cello tape.

To make a dodecahedron (12 faces) you have to make another identical bowl and glue the two together.

## Activity 58    QUADRILATERALS BY PAPER FOLDING.

*Materials:* 3 sheets of craft paper size 8 X 6 cm, scissors and glue.

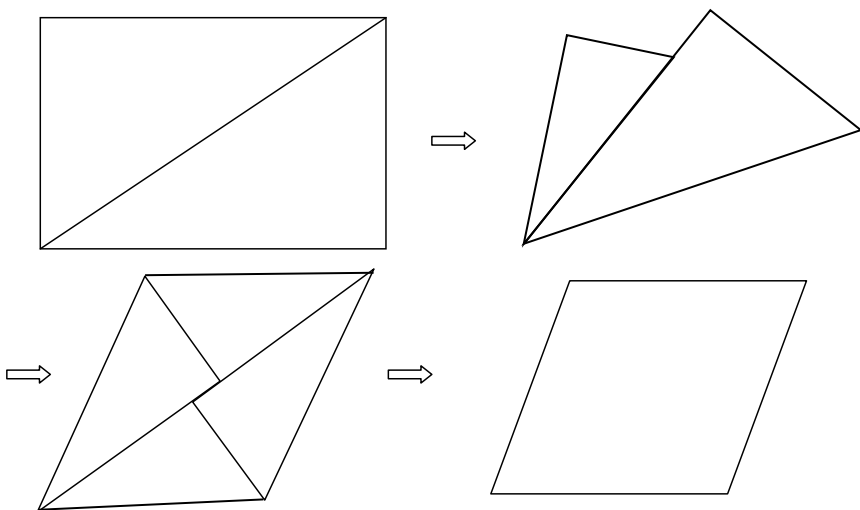
*Procedure*

### (1) Parallelogram.

Start with a rectangular sheet.

*Step 1* Fold the sheet diagonally (one diagonal), crease well and unfold.

*Step 2* Fold the top left vertex to the diagonal and then the bottom right vertex to the same diagonal as shown below. Then use scissors, cut and get a parallelogram.

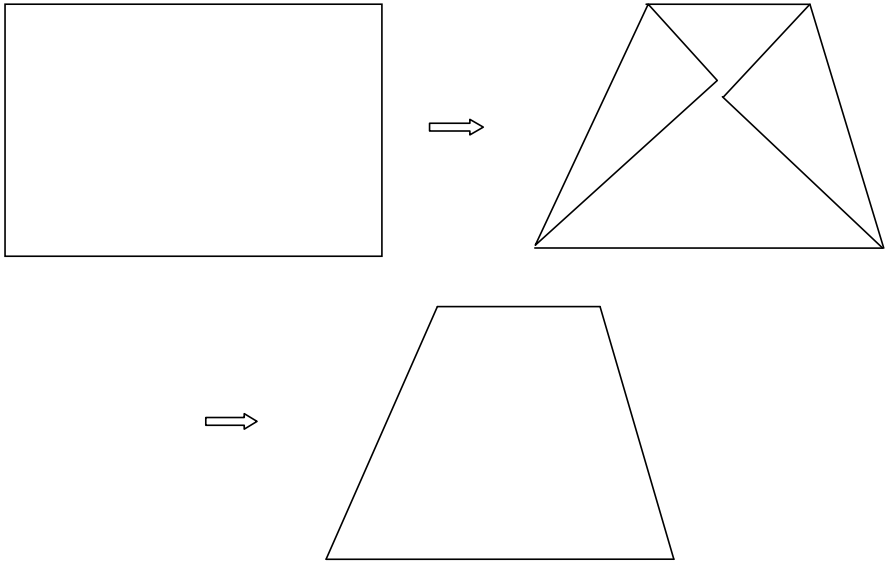


Stick the shape in your Activity sheet.

## (2) Trapezium

Start with a rectangular sheet.

Fold the top 2 corners as shown below, cut with scissors and get a trapezium.



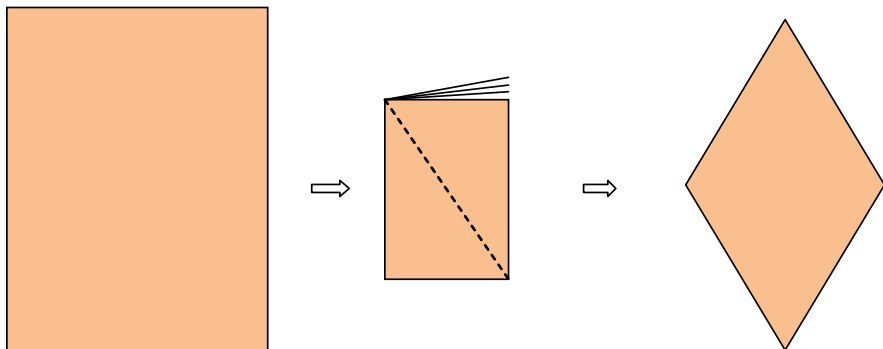
## (3) Rhombus

Start with a rectangular sheet. (Do not take a square sheet)

*Step 1* Fold into half lengthwise and then again into half. Do not unfold.

*Step 2* Fold the small rectangle (all the 4 layers) diagonally, make a good crease.

Unfold and cut, you will get a rhombus.



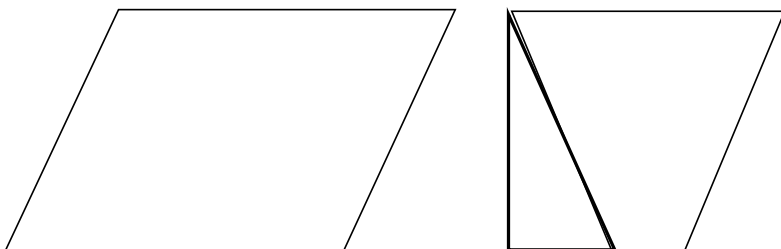
### Activity 59 AREA OF A PARALLELOGRAM BY PAPER FOLDING.

**Materials:** 2 sheets of rectangular craft paper of any convenient size, scissors.

**Procedure**

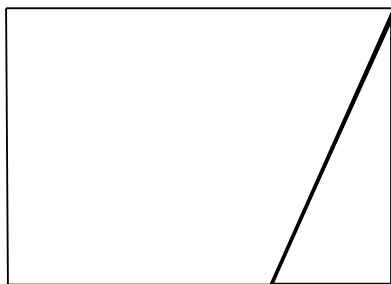
**Step 1** Make 2 identical parallelograms from 2 sheets.

**Step 2** Take one parallelogram. Fold the left bottom corner carefully onto the base and crease well. This crease will be the height of the parallelogram.



**Step 3** Take scissors and cut off the triangular part.

**Step 4** You will get a right triangle and a trapezium. Now place the triangle to the right of the trapezium in such a way that a rectangle is formed.



### *Step 5*

Length of this rectangle = base of the parallelogram =  $b$

Breadth of the rectangle = height of the parallelogram =  $h$

Therefore, area of the parallelogram =  $b \times h$

In the Activity Sheet, stick one parallelogram on the left and then the above rectangle to the right and write the details

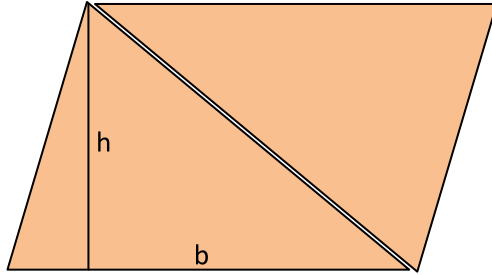
## **Activity 60 AREA OF A TRIANGLE USING A SHEET OF PAPER.**

*Materials:* 2 rectangular sheets of craft paper of any convenient size, scissors.

### *Procedure*

*Step 1* Make 2 identical scalene triangles from 2 sheets of paper.

*Step 2* Place the triangles as shown below.



*Step 3* You will get a parallelogram. Area of the parallelogram = base X height

Therefore area of the triangle =  $\frac{1}{2}$  (area of parallelogram)  
 $= \frac{1}{2}(b \times h)$

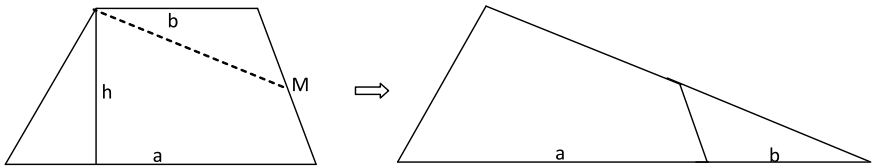
### Activity 61 AREA OF A TRAPEZIUM USING A SHEET OF PAPER

*Materials:* 2 rectangular sheets of craft paper of any convenient size, scissors.

*Procedure*

*Step 1* Make identical trapeziums with 2 sheets of paper.

*Step 2* Find the midpoint *M* of one of the non parallel sides and connect it to the opposite vertex as shown below.



*Step 3* Cut along the dotted line, you will get a triangle. When you place this triangle as shown in the 2<sup>nd</sup> figure, you will get a bigger triangle.

Area of this big triangle =  $\frac{1}{2}$  base  $\times$  height =  $\frac{1}{2} (a + b) \times h$   
 Therefore the area of the trapezium =  $\frac{1}{2} (a + b) \times h = \frac{1}{2} h (a + b)$

In the Activity sheet, stick one trapezium in the first box and the final triangle in the 2<sup>nd</sup> box with all the markings.

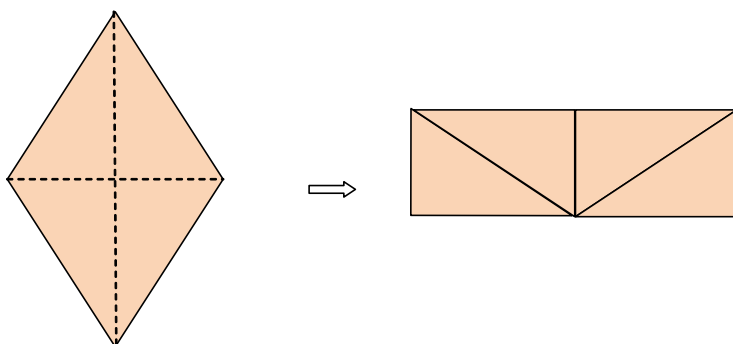
### **Activity 62 AREA OF A RHOMBUS USING A SHEET OF PAPER**

*Materials:* 2 rectangular sheets of craft paper of any convenient size, scissors.

*Procedure*

*Step 1* Make identical rhombuses with 2 sheets of paper.

*Step 2* Cut one of the sheets as shown below into 4 equal right triangles.



Step 3 Arrange the triangles as shown in the second figure as a rectangle.

Step 4 Mark  $d_1$  and  $d_2$  as the lengths of the diagonals. Then the length of the rectangle formed will be  $d_1$  while the breadth will be  $\frac{1}{2} d_2$ .

Therefore the area of the rhombus = area of the rectangle  
 $= d_1 \times \frac{1}{2} d_2$

$$= \frac{1}{2} d_1 d_2$$

In the Activity sheet, stick the rhombus on the left and the rectangle on the right.

### **Activity 63 COORDINATES - INTRODUCTION.**

*Materials:* Graph sheet

*Procedure:* Draw the x and y axes in the graph sheet taking the origin in the middle of

the sheet. Mark - 10 to 10 on both the axes.

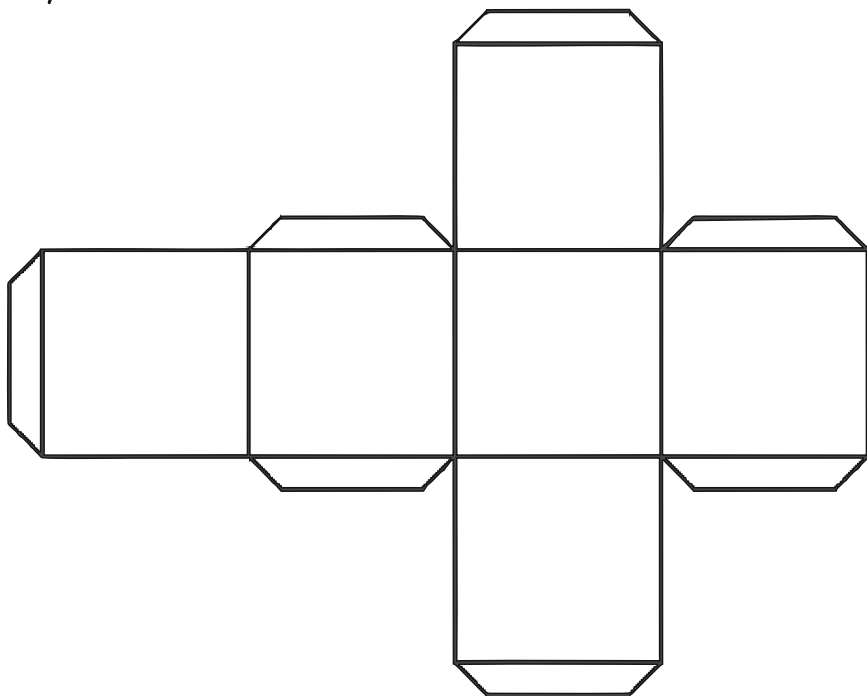
Take your Activity sheet and plot the points mentioned therein.

## **Activity 64 MAKE 3D SHAPES WITH CARDBOARD AND NETS**

*Materials:* KG cardboard of size 30X 30 cm (4 sheets), scissors, glue/cellotape

### **HOW TO MAKE A CUBE ?**

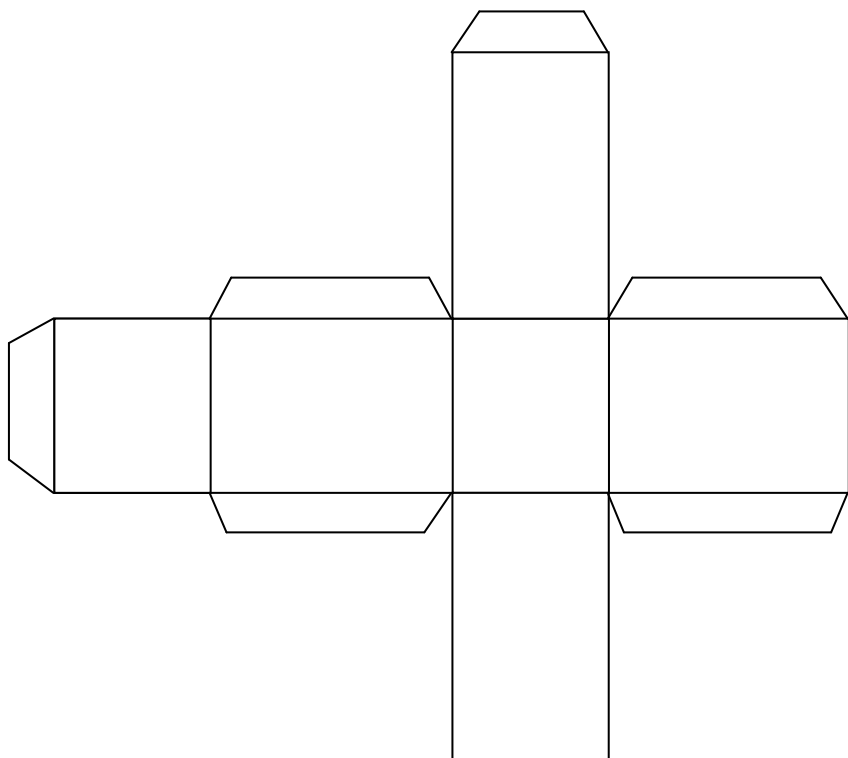
On one of the cards draw the following net accurately. You may take each side as 6 cm.



After drawing the net, cut carefully along the boundary, make good creases along all the inner lines and then fold it up to get a cube. Stick the stubs to get a good cube.

### **HOW TO MAKE A CUBOID?**

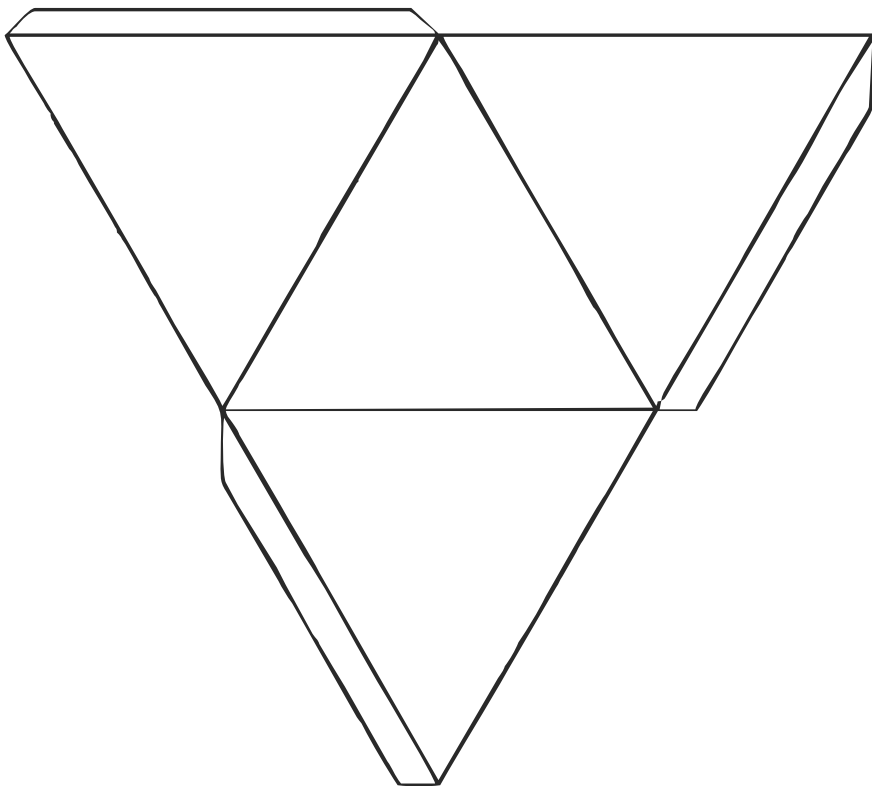
On one of the cards draw the following net accurately. You may take the dimensions as 8cm X 5cm X 5 cm



After drawing the net, cut carefully along the boundary, make good creases along all the inner lines and then fold it up to get a cuboid. Stick the stubs to get a good cuboid.

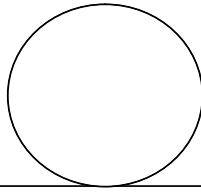
### **HOW TO MAKE A TETRAHEDRON (TRIANGULAR BASED PYRAMID)?**

On a card, draw the following net accurately. Cut along the boundary. Fold up, crease well and then stick the stubs with glue or cellotape. You may take each side as 10 cm.



### ***HOW TO MAKE AN OPEN CYLINDER?***

Draw the following net on a card , cut carefully along the boundary (you should get only one piece), roll the rectangular part so that the circular part becomes the bottom of the cylinder. Use glue/cellotape and stick the edges. Take the length of rectangle as 22 cm, breadth as 15 cm and the radius of the circle as 3.5 cm.



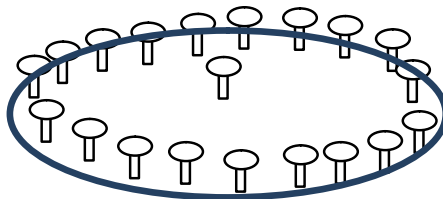
## **Activity 65    INTRODUCTION TO CIRCLES**

*Materials:* Circular Geo Board, rubberbands (4)

*Procedure*

*How to make a Geo Board (Peg Board)?*

Take a square plywood board of dimensions 15 cm X 15 cm and small screws (about 20) of length about 1 cm. Draw a circle of diameter 10 cm on the Board. Fix screws on the boundary of the circle at equidistant intervals after marking the points with a pencil. Fix one screw at the centre. Your Geo Board is ready!



1. Use the circular Geo Board and make a circle using a rubber band.
2. Measure its radius using a ruler.
3. Pass a rubber band around any two pegs on the circle so that a line segment is formed. This represents a chord.
4. Repeat the activity, we get different chords, some short, some long & some passing through the centre.
5. Measure the chords passing through the centre. What is your observation? They are the longest chords. A chord passing through the centre is **DIAMETER**.

*Measure its length. Verify that the diameter is twice the radius.*

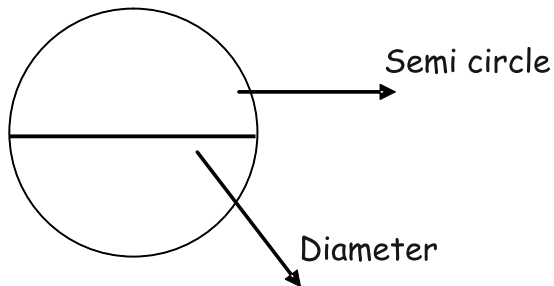
6. Using the geoboard and rubberbands get different parts of a circle namely, major and minor arcs, major and minor segments and sectors of various sizes.

## Activity 66 CIRCLE AND ITS PARTS BY PAPER FOLDING

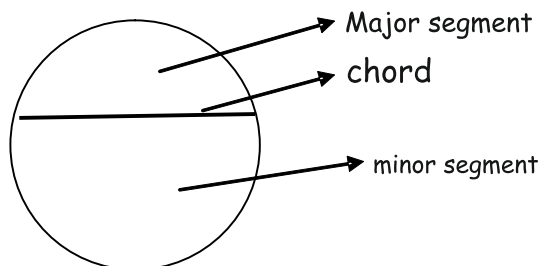
*Materials:* Circular sheets of craft paper (4) of diameter about 10 cm

### *Procedure*

- Fold one of the sheets into half, make a good crease and unfold to see a semi circle and a diameter. Stick the sheet in your Activity Book and label the parts.

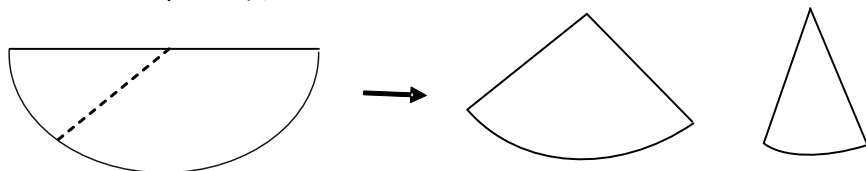


- Take another sheet, fold it into half, make a good crease and unfold, get one diameter. Fold the same sheet into half at a different angle and crease it well, get another diameter. Their point of intersection is the CENTRE of the circle. Stick the sheet in your Activity sheet with the name of each part.
- The 3<sup>rd</sup> sheet has to be folded to show a chord and segments.



- The 4<sup>th</sup> sheet has to be folded and cut to make sectors.

First fold into half and cut it into 2 pieces. Take only one of them.



Fold the semicircular sheet about the dotted line (radius) and cut it into 2 pieces.

You will get 2 sectors. Stick them in your Activity sheet and label them.

### Activity 67 DEFINITION AND CALCULATION OF PI

*Materials:*

(1) Some circular objects such as a cardboard disc, CD, bangle, plates, bottle caps, mugs etc

(2) thin measuring tape

*Procedure:*

- Measure the diameter of all the objects using a ruler
- Measure the circumference of all the objects in cm using a measuring tape and ruler

Enter your values in the Table printed in the Activity sheet. The values of  $c \div d$  in the last column should be nearly equal to 3.14. This value namely  $c/d$  is called **pi**, written as  $\pi$ .

No matter what the size of the circle is, circumference  $\div$  diameter is the same.

## Activity 68 Calculation of Circumference

**Materials:** Old CD, thin measuring tape, scissors.

**Perimeter** If you walk on a circle, the distance walked in a complete round is the perimeter or circumference of the circle.

Circumference of a circle is given by  $2\pi r$  or  $\pi d$  where  $r$  is radius and  $d$  is diameter.

**Procedure:**

- Make a small mark P at the edge of the CD with a marker/pen
- Draw a long line segment AB using a ruler.
- Place the disc vertically so that P falls on A.
- Roll the disc carefully on the line until P touches the line again at some point on the line. Call this point Q.
- Measure the length of PQ.
- Measure also the circumference of the disc using a thin measuring tape as accurately as possible.
- Write your observation.
- Measure the diameter of the CD as accurately as possible and calculate the product  $\pi \times d$  taking  $\pi$  as 3.14.
- All the 3 values should nearly be equal.

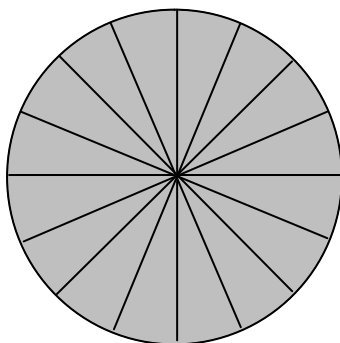
## Activity 69 AREA OF A CIRCLE.

*Materials:* A circular cardboard disc of diameter about 12 cm and scissors

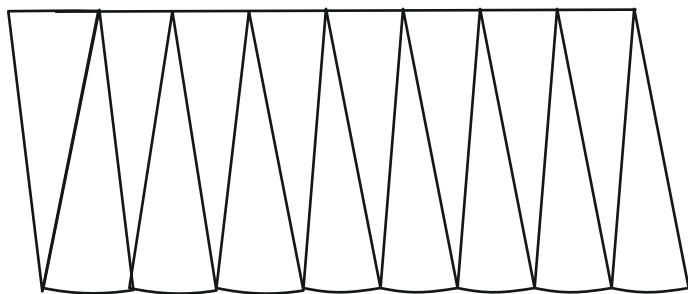
Area is the space occupied by a circle on a plane and it is calculated by the formula  $\pi r^2$

This formula can be justified in the following activity.

- Draw 8 diameters as shown below and cut along all the diameters. You will get 16 pieces, each piece being a sector of the circle. For better results you may draw more diameters and get more sectors.



- Stick all the sectors side by side as shown below.



- When all the pieces are arranged, the figure becomes approximately a parallelogram.
- Sum of lengths of the top and bottom of the above parallelogram =  $2\pi r$  (circumference)

Therefore, the base can be taken as  $\pi r$  and *approximate* height as  $r$ .

$$\begin{aligned}\text{Area of the parallelogram} &= \text{Base} \times \text{Height} \\ &= \pi r \times r = \pi r^2\end{aligned}$$

Therefore the area of the circle = area of the parallelogram  
 $= \pi r^2$

Note: This is NOT the proof of the formula, it is only a demonstration.

Conclusion

Area of circle = .....

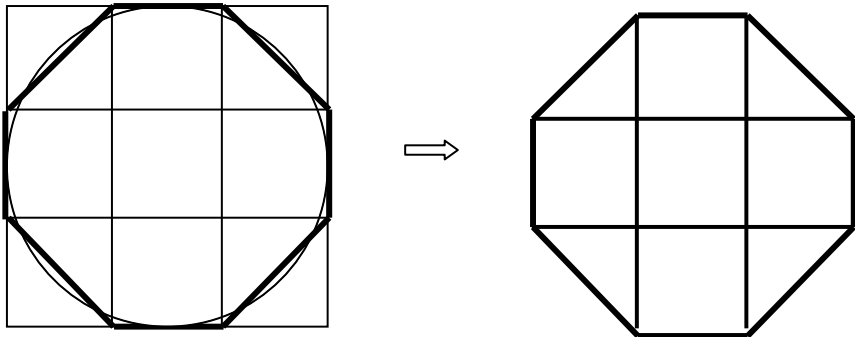
## Activity 70 AREA OF A CIRCLE BY EGYPTIAN METHOD (without the concept of pi).

Materials: 9 X 9 cm cardboard and scissors.

Procedure

- Divide the square card into 9 small squares by drawing 4 suitable lines
- On the cardboard draw a circle just touching all the 4 sides of the card.
- Draw the diagonals of the 4 corner squares as shown.
- With scissors, cut out the octagon obtained.

Stick the card in your Activity Book in the space provided and do the calculations as shown below.



- Area of each small square is  $(1/9) A$ , where  $A$  is the area of the original square and the 4 corners (triangular) together is of area  $(2/9) A$
- Total area =  $(2/9)A + (5/9)A = (7/9)A$

But  $A = 9 \times 9 = 81 \text{ cm}^2$  and hence area of the circle =  $(7/9) \times 81 = 63 \text{ cm}^2$  (approx)

(Verification: Using area formula we get  $\pi \times 4.5 \times 4.5 = 63.585$  (approx))

## MISCELLANEOUS ACTIVITIES

### Activity 71 PARABOLIC CALCULATOR

Materials: Graph sheet and a thin straw/stick

#### Procedure

In this activity, CALCULATOR does not mean an electronic or other calculators. This is just a geometrical method of calculating products, quotients and squares and square roots of numbers using a parabola.

Parabola is a U-shaped curve which can be obtained by plotting the graph of the function  $y = x^2$ .

*Step 1* Construct a table of values of  $x$  and  $y$

x	0	±1	±2	±3	±4	±5	±6
y	0	1	4	9	16	25	36

*Step 2* Plot these points on a graph sheet. Take a scale of 1 unit = 1cm on the

x-axis and 1 cm = 5 units on the y- axis. Graph is shown below for reference.

Note that minus signs are not written on the x-axis to its left. *See next page for figure.*

How to use?

*Illustration 1* Multiply 4 by 5

Connect the points ON THE CURVE exactly above 4 and 5 on the x-axis with a thin stick and see where the stick crosses the y axis. The point at which the stick crosses the y axis will give the answer 20.

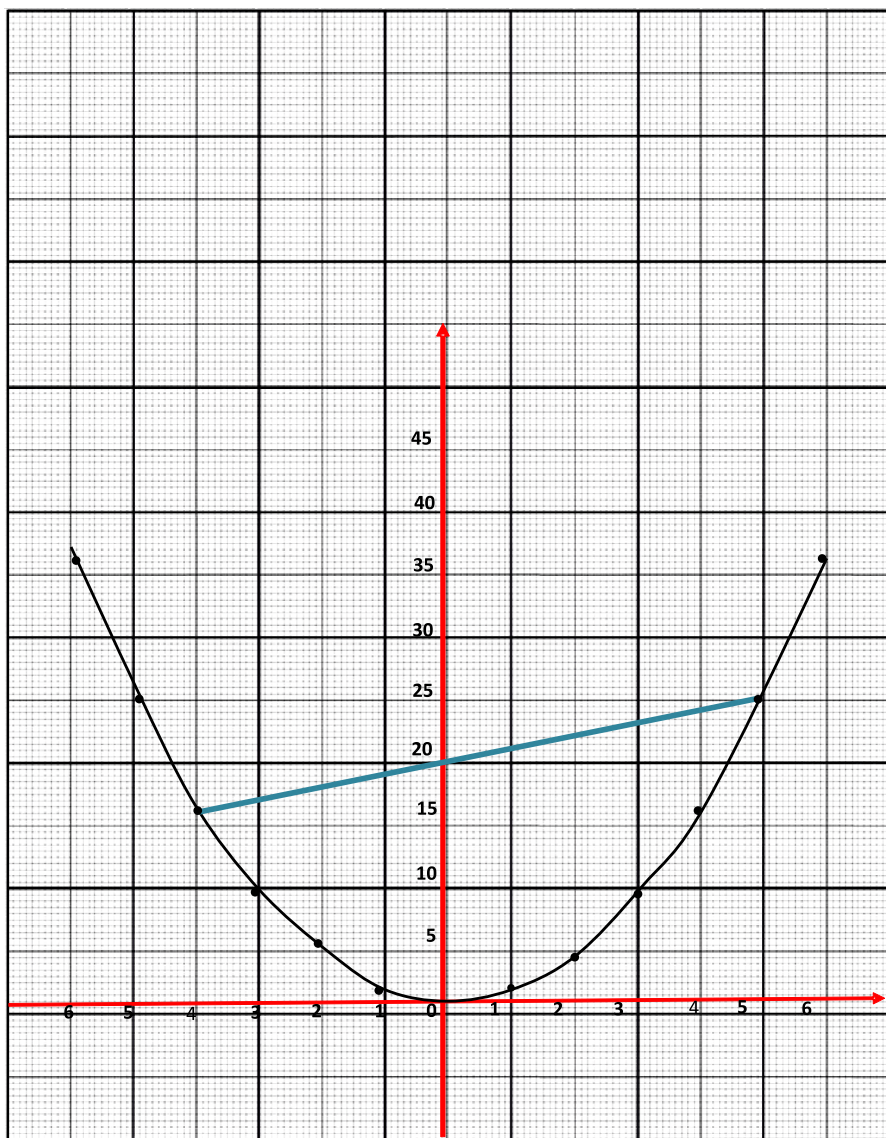
*Illustration 2* Divide 20 by 4.

Connect the points 20 on the **y axis** and 4 on the **x axis** with a stick. The point where the stick crosses the parabola will give the quotient 5.

*Illustration 3* Find the square root of 36

Place a stick at 36 on the **y axis** horizontally and see where it cuts the curve. The **x coordinate** of this point is the answer. There will be 2 answers 6 and - 6.

*Note that the values obtained will only be approximate because the parabola drawn may not be accurate.*



## Activity 72 FINGER MULTIPLICATION

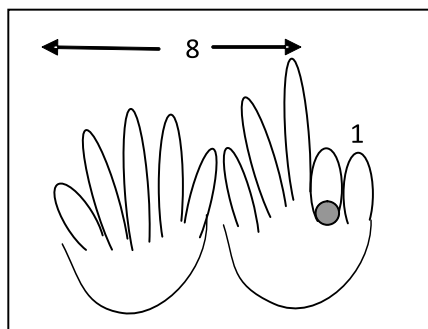
Most of you would have used your fingers at one time or other, for counting and adding. Can the fingers be used for multiplying numbers?

Fingers can be used for multiplying **one digit and two-digit numbers by 9**.

### Procedure

Spread all the 10 fingers, hands facing you. The first finger on your left hand (that is, the thumb) is counted as 1, the next finger as 2 and so on till you end up with 10 which is the thumb on your right hand. In other words count always *from left to right*.

To multiply 9 by 9, fold the 9<sup>th</sup> finger first. Now you find 8 fingers to the left of the folded finger and 1 finger on the other. The answer is 81. This is shown below.



You may try  $9 \times 2$ ,  $9 \times 3$  etc.

If you want to multiply a single digit number by 99, the same procedure works with a small modification. For example, to multiply 6 by 99, fold the 6<sup>th</sup> finger, then you

will see 5 upright fingers to the left of the folded finger and 4 on the right. Read the answer as 594, *reading the folded finger as 9. This works for all digits. Furthermore to multiply by 999, read the folded finger as 99 and repeat the procedure.*

**Multiplication of a 2-digit number by 9:** To start with you should spread all your fingers in front of you so that you will be able to count from 1 to 10.

Let us take a number having its **second digit bigger than the first**.

**Multiply 47 by 9:**

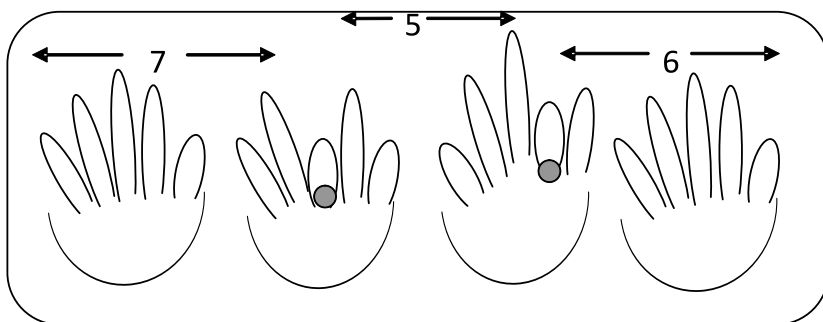
Spread all the fingers in front of you. *Group* the first 4 fingers (imagine a rubberband round these 4 fingers) together. Number of grouped fingers namely 4 is the first digit of the answer. Now fold down the 7<sup>th</sup> finger. There are 2 upright fingers next to this group. So the second digit of the answer is 2. Finally there are 3 upright fingers to the right of the folded finger. Hence the last digit of the answer is 3. Answer is **423**.

***Suppose the second digit is less than the first, then we proceed as follows.***

**Multiply 84 by 9**

First of all, you have to 'borrow' the hands of someone next to you sitting on your left so that there will be 20 fingers to work with. Both of you should spread your hands in front of your faces so that you will be able to count from 1 to 20, left to right.

The person on your left should fold the 8<sup>th</sup> finger down while you fold your 4<sup>th</sup> finger down. Now read the answer as 756.



*Suppose the two digits are equal, then we proceed as follows.*

Multiply 77 by 9 (which is same as multiplying 7 by 99)

After spreading all the fingers, fold the 7<sup>th</sup> finger down. Now there will be 6 fingers to the right of the folded finger and 3 to the right. Read the *folded finger as 9*, we get the answer as 693.

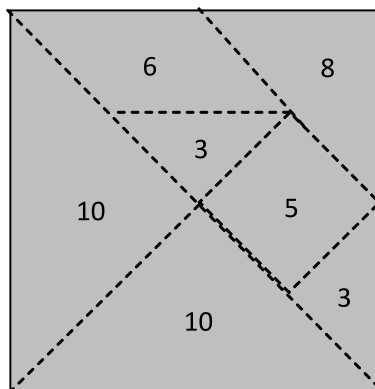
*Note that all the above methods work only for 9.*

### **Activity 73 TANGRAM puzzle**

*Tangram is a square cut into various pieces which can be rearranged into different shapes.*

**Materials:**

1. A square piece of cardboard
2. Scissors
3. Ruler and pencil



Take a square piece of card and draw lines exactly as shown. With scissors, cut carefully along the dotted lines and get 7 pieces.

You will have 5 triangular pieces, a parallelogram and a square.

Write numbers 10, 8, 6, 5 and 3 at the centre of the pieces as shown. They represent the **values** of the respective pieces. The total value of all the pieces is **45**.

**You are given 8 questions below.**

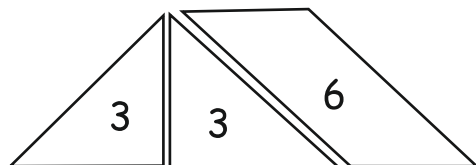
1. Make a square worth 20 using 2 pieces.
2. Make a square worth 6 with 2 pieces.
3. Make a rectangle worth 11 using 3 pieces.
4. Make a trapezium worth 12 with 3 pieces.
5. Make a trapezium of value 18 using 2 pieces.
6. Make a parallelogram of value 20 using 2 pieces.
7. Make pentagon worth 17 using 3 pieces.
8. Make a pentagon worth 24 with 3 pieces.

As an illustration let us take **Question No 4**

Since the value has to be 12 we have to choose 3 pieces whose total value is 12.

We can get a trapezium in many ways but any 3 pieces may not fit together

resulting in a trapezium. We can choose 2 right angled triangles and the parallelogram and arrange them as follows.



This is a trapezium of value 12 and so it is the answer.

## Activity 74 MULTIPLICATION MADE EASY I (2- digit numbers)

Using this method you can multiply two numbers very quickly in *one step*.

Remember, it will be easy *only after practice*.

**Illustration 1** Multiply 21 by 32

**Step 1** Multiply 1 by 2 and write it  
in the units place.  
Let us call this 'Direct Product'

$$\begin{array}{r} 21 \\ \times 32 \\ \hline 2 \end{array}$$

Thus, Direct Product = 2

**Step 2** Calculate  $(2 \times 2) + (1 \times 3)$   
 $= 4 + 3 = 7$

Let us call this 'Cross Product'  
Cross Product = 7

$$\begin{array}{r} 21 \\ \times 32 \\ \hline 32 \end{array}$$

Write 7 in the tens place of the answer.

**Step 3** Multiply 2 by 3  
Second Direct Product = 6  
Write 6 in the hundreds place  
of the answer.  
Answer = 672

$$\begin{array}{r} 21 \\ \times 32 \\ \hline 672 \end{array}$$

**Illustration 2** Multiply 45 by 27

**Step 1** Direct Product =  $5 \times 7$   
 $= 35$   
Write 5 in the units place  
of the answer,

$$\begin{array}{r} 45 \\ \times 27 \\ \hline 5, \\ \text{carry } 3 \end{array}$$

$$\begin{array}{r} 4 \quad 5 \\ \times \quad \times \\ 2 \quad 7 \\ \hline 1 \quad 5 \end{array}$$
$$\begin{array}{r} 4 \quad 5 \\ \hline 2 \quad 7 \end{array}$$

---

1    2    1    5

## Activity 75 MULTIPLICATION MADE EASY II

### (3- digit numbers)

The procedure can be remembered as follows.



*Illustration* Multiply 513 by 123

$$\begin{array}{r} 513 \\ \times 123 \\ \hline 9 \end{array}$$

*Step 1* First direct product =  $3 \times 3$

$\therefore$  Units digit of the answer is 9

*Step 2* First cross product =  $(1 \times 3) + (3 \times 2)$   
 $= 3 + 6 = 9$

$$\begin{array}{r} 513 \\ \times 123 \\ \hline 99 \end{array}$$

$\therefore$  Tens place of the answer = 9

*Step 3* Mixed product =  $(5 \times 3) + (3 \times 1) + (1 \times 2)$   
 $= 15 + 3 + 2$   
 $= 20$

Hundreds place = 0, with a carry 2

$$\begin{array}{r} 513 \\ \times 123 \\ \hline 099 \\ \text{carry } 2 \end{array}$$

*Step 4*    Second cross product =  $(5 \times 2) \times (1 \times 1)$

$$= 10 + 1$$

$$= 11$$

Add the carry 2 and get  $11 + 2 = 13$

Thousands place = 3, with a carry 1

$$\begin{array}{r} 5 \ 1 \ 3 \\ \times \phantom{0} \\ \hline 1 \ 2 \ 3 \\ 3 \ 0 \ 9 \ 9 \\ \hline \text{carry } 1 \end{array}$$

*Step 5*    Second direct product =  $5 \times 1$

$$= 5$$

Add the carry 1 and get  $5 + 1 = 6$

$$\begin{array}{r} 5 \ 1 \ 3 \\ \downarrow \\ 1 \ 2 \ 3 \\ \hline \end{array}$$

Enter 6 and get the answer as **63099**

$$\begin{array}{r} 6 \ 3 \ 0 \ 9 \ 9 \\ \hline \end{array}$$

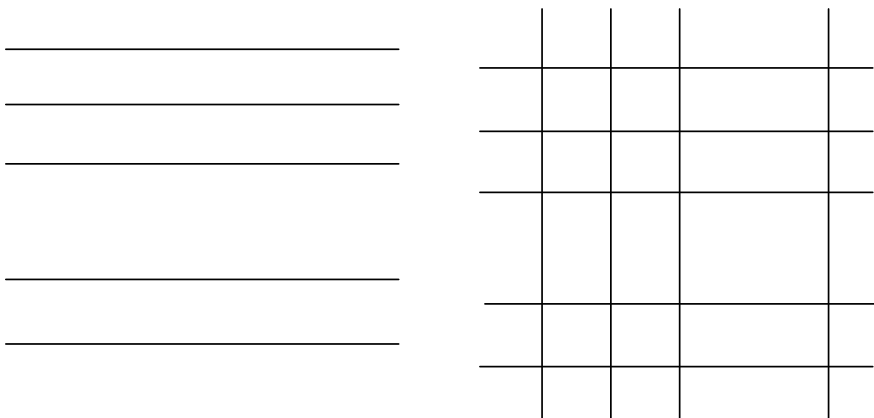
*Note: Once you master the technique, it is possible to extend it to any number of digits.*

## Activity 76    **MAGIC MULTIPLICATION**

In this Activity, you will learn to multiply two numbers just by *counting dots*!

### *Illustration 1*    Multiply 32 by 31

**Step 1**    Represent 32 by drawing 3 horizontal lines of any convenient length followed by 2 more lines with a gap in the middle as shown in the *first* diagram below. You *need not* use a ruler to draw lines. Just freehand drawing will do. Accuracy is not required in any of the drawings.

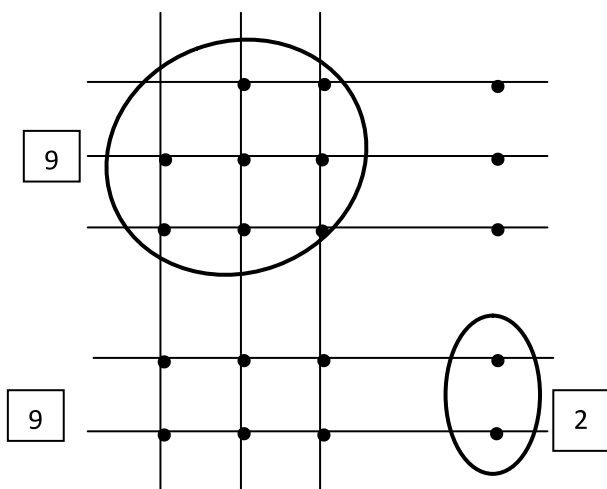


**Step 2**    Represent 31 by drawing 3 vertical lines followed by 1 more vertical line with a gap in the middle as shown in the *second* diagram, to intersect the horizontal lines.

**Step 3**    Mark all the points of intersection by bold dots.

**Step 3**    Draw 2 freehand circles or ovals round the clusters of points in 2 opposite corners as shown below. Now there are 3 regions.

**Step 4**    Count the number of dots in each region.



Number of dots in the bottom most part gives the units place of the answer. Number of dots in the middle portion gives the tens place and the number of dots in the upper portion is the hundreds place.

**Answer = 992**

*Illustration 2* Find the value of  $213^2$

*Step 1* Draw 2 lines of any convenient length horizontally, leave some space and then draw 1 line parallel to these. Now leave some space and draw 3 lines horizontally below.

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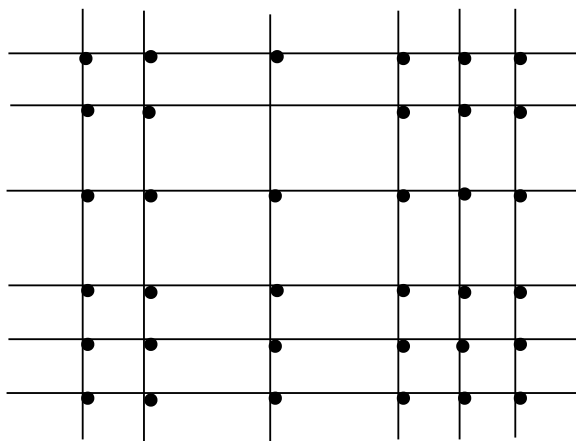


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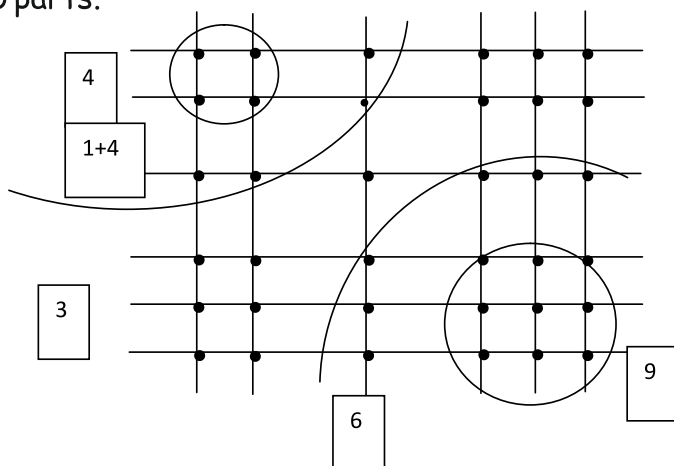


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*Step 2* Draw 2 lines vertically, leave some space and then draw 1 more vertical line of the same length. Leave some space and draw 3 vertical lines to intersect the above lines. Mark all the points of intersection by dots.



*Step 3* Draw 2 freehand circles or ovals round the clusters of points in one pair of opposite corners as shown below. Draw a curve just below the top circle and another curve just above the bottom circle. Thus the whole grid is divided into 5 parts.

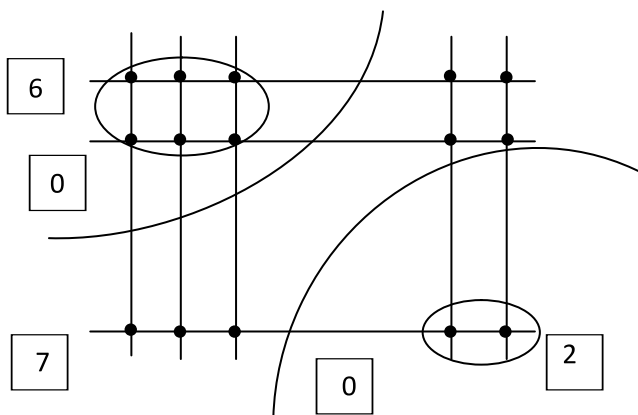


**Step 4** Count the number of dots in each region. There are 9 dots in the lowermost region. So the units place of the answer is 9. In the next higher region there are 6 dots. So the tens place is 6. Moving upwards, there are 13 dots in the middle region and hence the hundreds place of the answer is 3, with a carry 1. The number of dots in the next higher region is 4 and adding the carry 1, thousands place becomes 5. Finally the ten thousands place must be 4 since there are 4 dots in the uppermost region.

Therefore the **answer is 45369**

*Note 1 If a digit is 0, the corresponding space has to be left blank without any line.*

Example:  $201 \times 302$



**Answer is 60702**

*Note 2 Magic multiplication may become messy if the digits in the given numbers are 5 or more.*

## Activity 77 SQUARE ROOT OF A SQUARE NUMBER

You know that 1 is the square root of 1 because  $1^2 = 1$ , 2 is the square root of 4 because  $2^2 = 4$ . Similarly square root of 9 is 3, square root of 16 is 4 and so on.

You also know that square root of a square number can be found by factoring the number. Another method of finding square root is the *division method*.

This Activity will teach you to find square roots by successive subtraction..

*Illustration* Find the square root of 36.

*Method:* Subtract 1, 3, 5, 7 and so on (all the odd integers) successively till you get 0.

$$\begin{array}{r} 36 \\ - 1 \\ \hline 35 \\ - 3 \\ \hline 32 \\ - 5 \\ \hline 27 \\ - 7 \\ \hline 20 \\ - 9 \\ \hline 11 \\ - 11 \\ \hline 0 \end{array}$$

Count the number of minus signs. Since there are 6 minus signs, the answer is 6.

Thus square root is just the number of subtractions.

This activity can be done using a strip of paper as follows.

Take a strip of paper of width about 2 cm. Divide the strip into 36 equal parts by drawing equally spaced lines. Alternatively, a ruled sheet may be used.



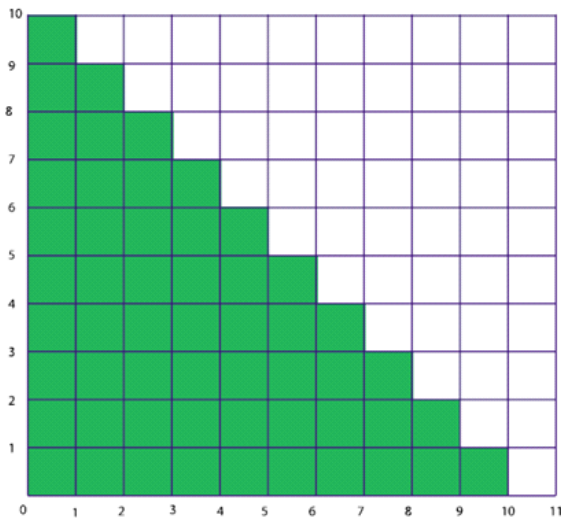
Take a pair of scissors and cut off one box from the right end, next cut off 3 boxes from the same end, then 5 boxes, 7 boxes, 9 boxes and 11 boxes. Count the number of pieces that fell down. 6 pieces fell down. Therefore 6 is the square root of 36.

**Activity 78** To verify that the sum of first  $n$  natural numbers is  $\frac{n(n + 1)}{2}$

**Materials:** Grid sheet of size 11 X 10 cm, color pencil, scissors

Assume that each box is a unit square.

With a color pencil shade the first box, then shade 2 boxes below it, next shade 3 boxes below and so on till you go the last row in which 10 boxes have to be shaded.



**Total area of shaded part = Sum of natural numbers from 1 to 10 =  $1+2+3+4+5+6+7+8+9+10$**

Now, cut the shaded portion and place it on the remaining part of the grid.

It will be found that it completely covers the grid.

Area of the whole squared paper is  $10 \times 11 \text{ cm}^2$ . Area of the shaded portion is  $\frac{(10 \times 11)}{2}$

This can be generalized to  $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$

**Activity 79 TO VERIFY THAT THE SUM OF FIRST  $n$  ODD NUMBERS IS  $n^2$**

*Materials:* 10 X 10 grid sheet, color pencil set

Each cell is 1 unit.

Step 1 Shade one cell in one corner of the sheet.

Step 2 Shade next 3 cells as shown .

$$\therefore \text{Total area of cells shaded} = 1 + 3 = 4 = 2^2$$

Step 3 Shade next 5 cells as shown .

$$\therefore \text{Total area of cells shaded} = 1 + 3 + 5 = 9 = 3^2$$

Step 4 Shade next 7 cells as shown.

$$\text{Total area of cells shaded} = 1 + 3 + 5 + 7 = 16 = 4^2$$

In general, if you continue shading cells in the same manner, you should get

$$1 + 3 + 5 + 7 + 9 + \dots + n = n^2$$

## Activity 80 Construction of MAGIC SQUARES

No specific material is required.

*Magic Square is a square arrangement of numbers in such a way that no matter*

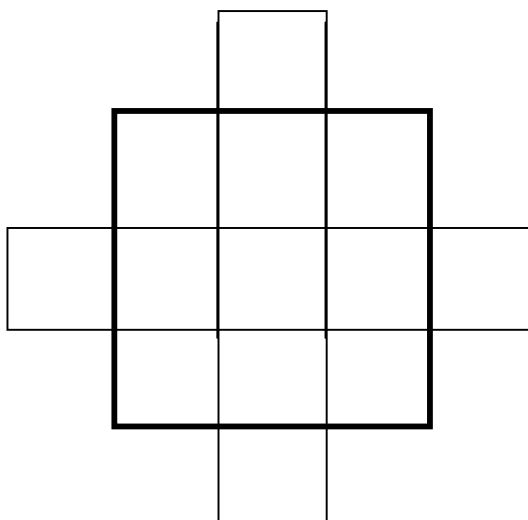
*how we add the numbers, along a row or column or diagonally, we get the same sum.*

*This sum is called the 'magic sum'.*

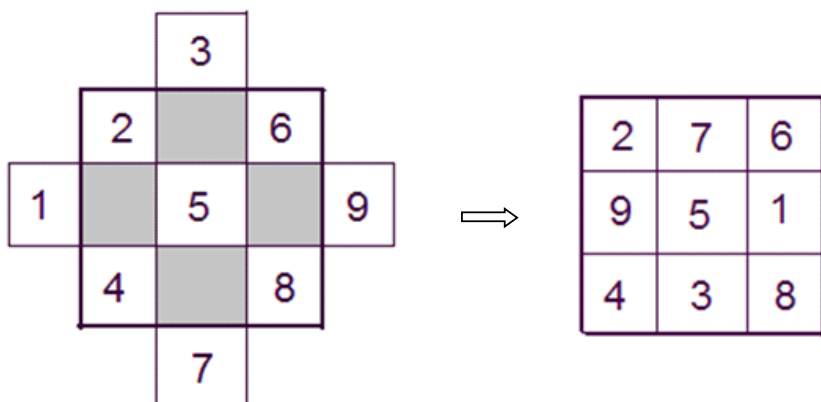
In this Activity you can learn how a 3 X 3 magic square can be constructed easily.

We shall use the numbers 1 to 9 to fill the boxes.

**Step 1** Prepare a 'grid' as follows. There are 4 **extra boxes** outside the 3 X 3 square.



**Step 2** Enter the numbers 1, 2, 3 along the top 'diagonal' (white boxes) as shown below. Enter the numbers 4, 5, 6 along the middle 'diagonal'. Enter the numbers 7, 8, 9 along the bottom 'diagonal'.



*Step 3* The numbers 1, 3, 7 and 9 are outside the square. Shift them to the dark boxes opposite (not adjacent) to them. Erase the unwanted boxes. Magic square will be ready!

*Note:* After some practice, you don't have to draw the outside squares. Just imagine them and start writing along the 'diagonals' and complete it.

**MAGIC SQUARE OF ORDER 4** It easy to construct a 4 X 4 magic square as follows.

x			x
	x	x	
	x	x	
x			x

- Put a small x mark in the boxes along the 2 diagonals
- Start from the first square in the first row and recite 1, 2, 3 etc upto 16 and fill the empty boxes (without x mark) from the first to the last box

x	2	3	x
5	x	x	8
9	x	x	12
x	14	15	x

- Start from the first square in the first row and recite 16 15, 14, 13, etc to 1 backwards and fill only the boxes with x mark. Erase all the 'x' marks, your magic square will be ready!

x 16	2	3	x 13
5	x 11	x 10	8
9	x 7	x 6	12
x 4	14	15	x 1

### 3 X 3 Magic square having numbers other than 1 to 9

Step 1 Write any number in the middle box

Step 2 Choose a magic sum as 3 times the middle number

Step 3 Write any other 2 numbers in any other 2 boxes.

Step 4 Fill the other boxes easily using the magic sum chosen by you. (some numbers may be repeated)

Example

- Write 9 in the middle box.
- Take magic sum as 27
- Write 7 and 10 in any other 2 boxes

Complete the magic square easily!

<div>10</div>	5	12
11	<div>9</div>	<div>7</div>
6	13	8